

# 3d mirror symmetry for characteristic classes & bow varieties

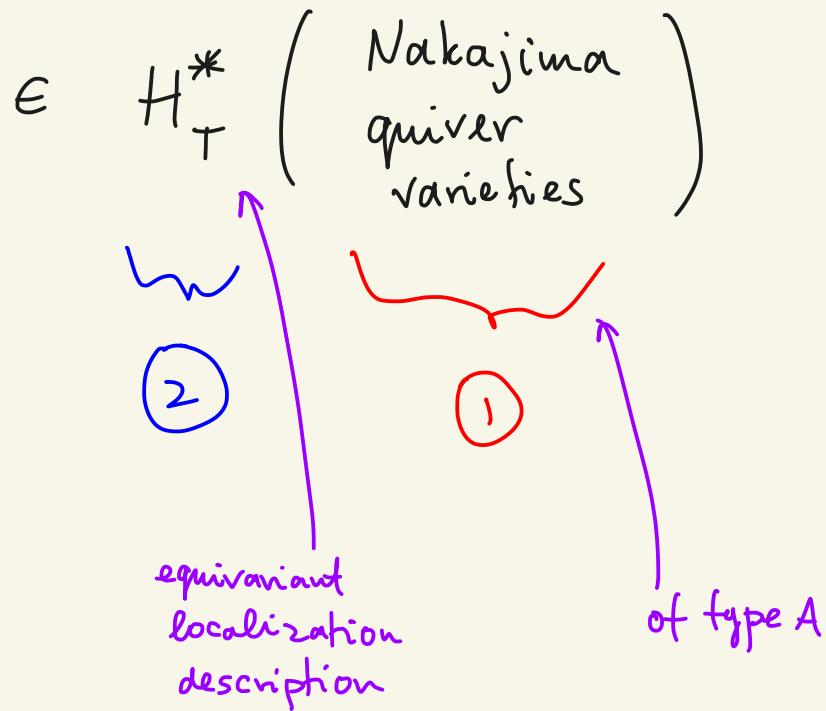
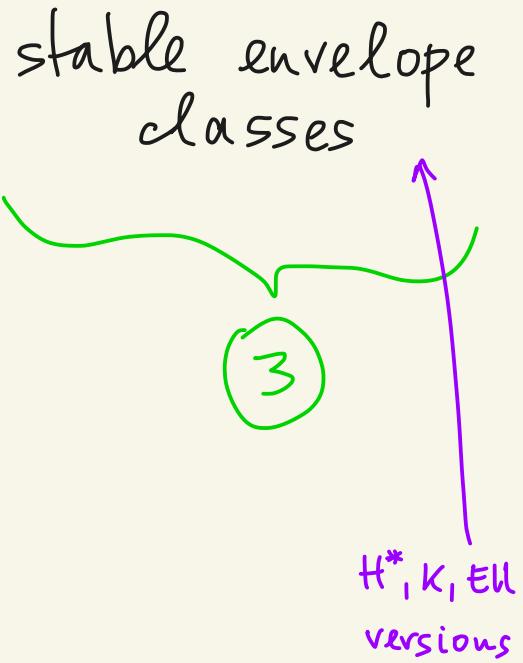
Richárd Rimányi  
UNC Chapel Hill

M-seminar  
April 1  
2021

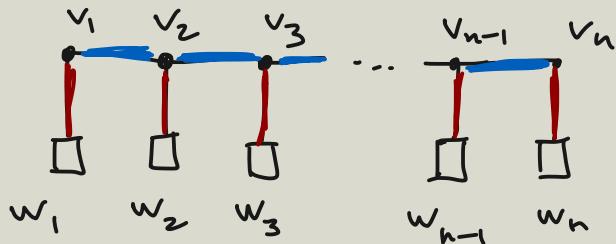


- joint work with Yitian Shou
- learned about branes from Lev Rozansky
- related works with
  - Andrey Smirnov
  - Alexander Varchenko
  - Zijun Zhou
  - Andrzej Weber

First goal :



# Nakajima quiver varieties:



quiver  $Q$

$\mathcal{N}(Q)$

quiver variety

Ex

$$\mathcal{N}\left(\begin{smallmatrix} k & \bullet \\ n & \square \end{smallmatrix}\right) = T^* \mathrm{Gr}_k \mathbb{C}^n$$

$$\mathcal{N}\left(\begin{smallmatrix} k_1 \leq k_2 \leq k_3 \\ \bullet - \bullet - \bullet \\ n \end{smallmatrix}\right) = T^* \mathcal{F}_{k_1, k_2, k_3, n}$$

$$\mathcal{N}\left(\begin{smallmatrix} 1 & 1 \\ \square & \square \\ 1 & 1 \end{smallmatrix}\right) = \widetilde{\mathbb{C}^2 / \mathbb{Z}_3}$$

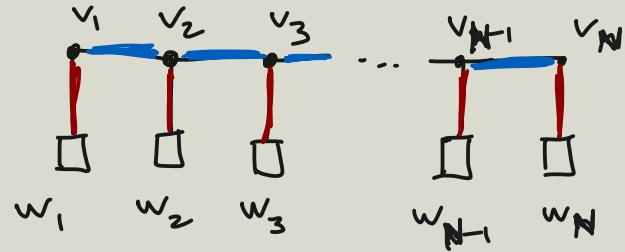
def

$$\mathcal{N} \left( \begin{array}{ccccccc} v_1 & v_2 & v_3 & & v_{n-1} & v_n \\ \downarrow l & \downarrow k & \downarrow b & \cdots & \downarrow a & \downarrow \\ w_1 & w_2 & w_3 & & w_{n-1} & w_n \end{array} \right) = \cdots$$

- $R := \bigoplus_i \text{Hom}(\mathbb{C}^{v_i}, \mathbb{C}^{v_{i+1}}) \oplus \bigoplus_i \text{Hom}(\mathbb{C}^{v_i}, \mathbb{C}^{w_i})$
  - $\mu: R \oplus R^* \rightarrow \bigoplus_i \text{End}(\mathbb{C}^{v_i})$        $\mu = [a, b] - ek$
  - $N(Q) := \tilde{\mu}^*(0)^{ss} / \bigtimes_i GL_{v_i}$

$N(Q)$

(type A)



- smooth
- holomorphic symplectic
- $T = (T^{w_1} \times T^{w_2} \times \dots \times T^{w_N}) \times \mathbb{C}_{\hbar}^*$  action
- finitely many fixed pts
- "tautological"  $v_1, v_2, \dots, v_N$ -bundles

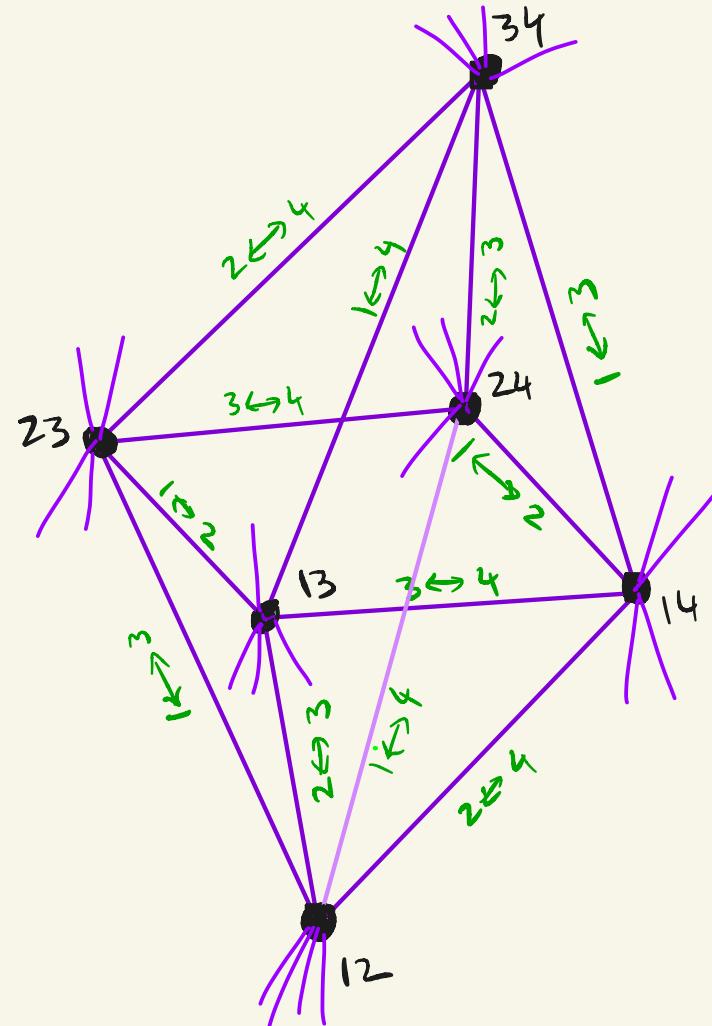
$$H_T^*(N(Q)) = ?$$

$$\bigoplus_{P \text{ T-fix}} H_T^*(P) \quad C[z_1, \dots, z_n, t]$$

Localization map, Loc

$$\text{im}(Loc) = ?$$

constraints among the components

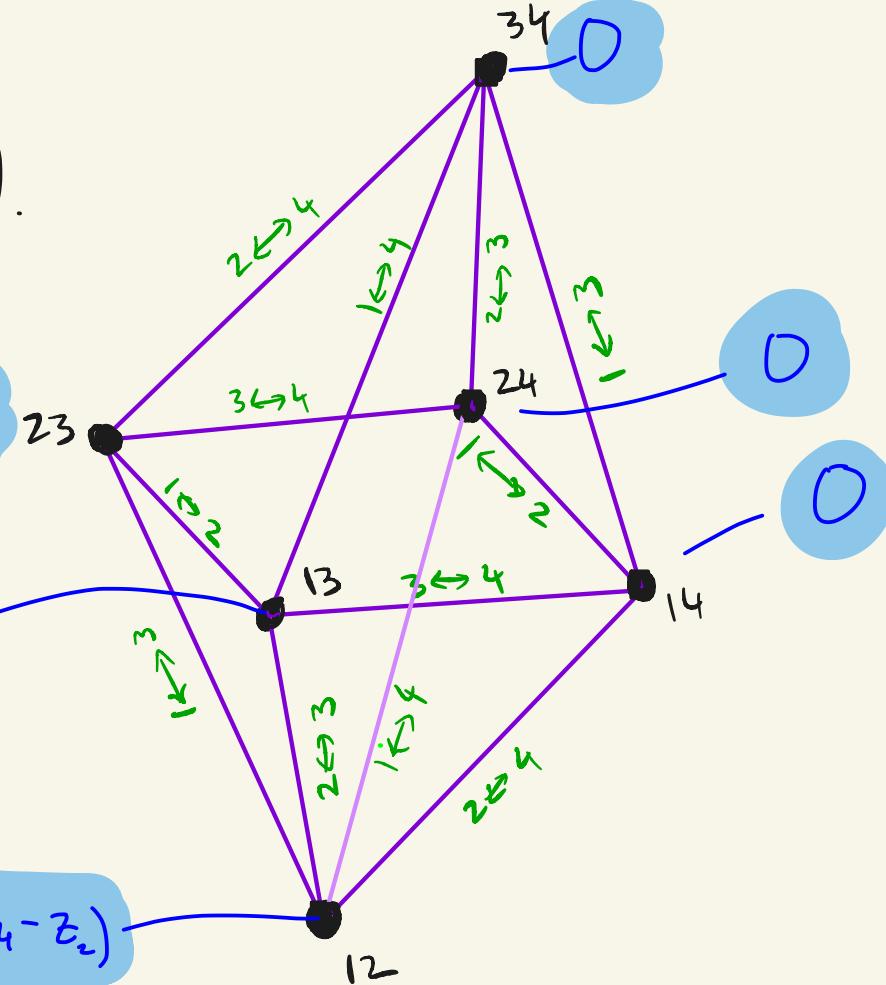


For example, this  
6-tuple is an  
element of  $H_T^*(\mathrm{Gr}_2 \mathbb{C}^4)$ .

$$(z_4 - z_3)(z_4 - z_2)$$

$$(z_4 - z_1)(z_4 - z_3)$$

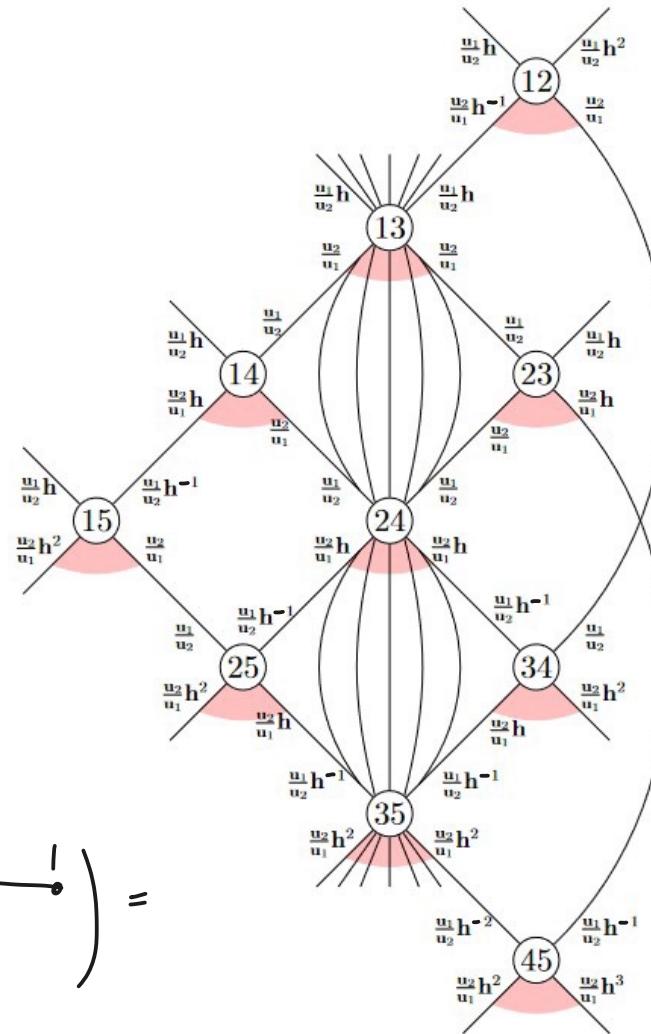
$$(z_4 - z_1)(z_4 - z_2)$$



## Warning

- $T^* \text{Gr}_2 \mathbb{C}^4$  was special ("GKM")
- In general the constraints among components are more restrictive

$$\mathcal{N}\left(\begin{array}{c|ccccc} & & & & \\ \bullet & 2 & 2 & 1 & & \\ & | & | & | & & \\ & \square & \square & \square & & \\ & | & | & | & & \\ & 1 & 1 & 1 & & \end{array}\right) =$$

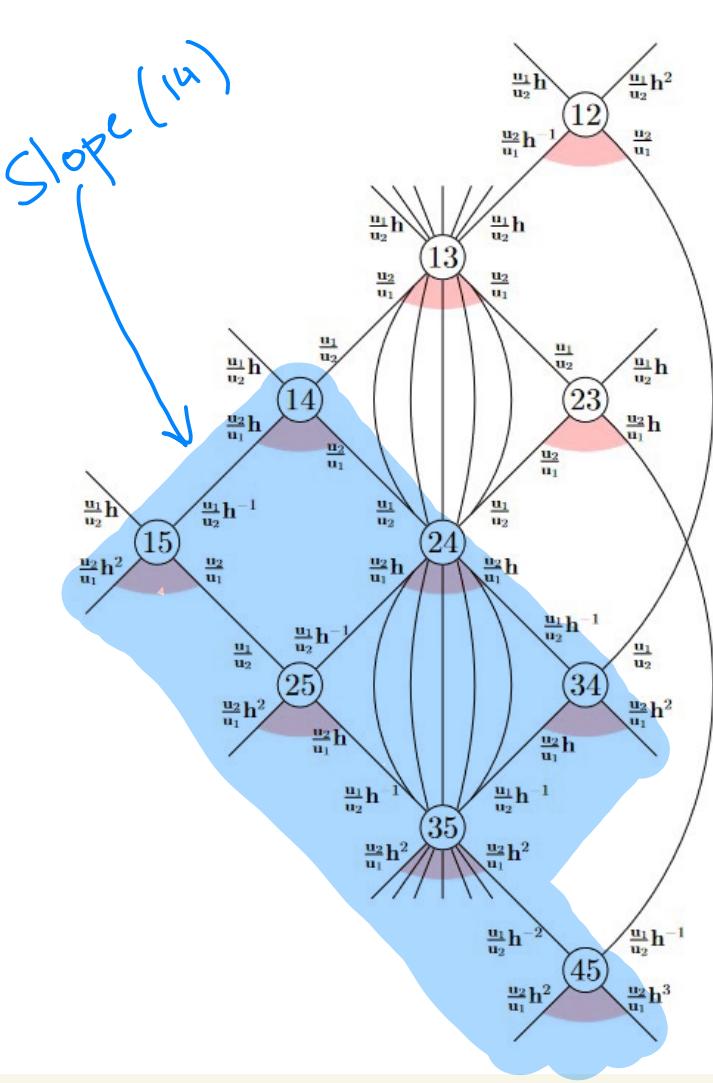
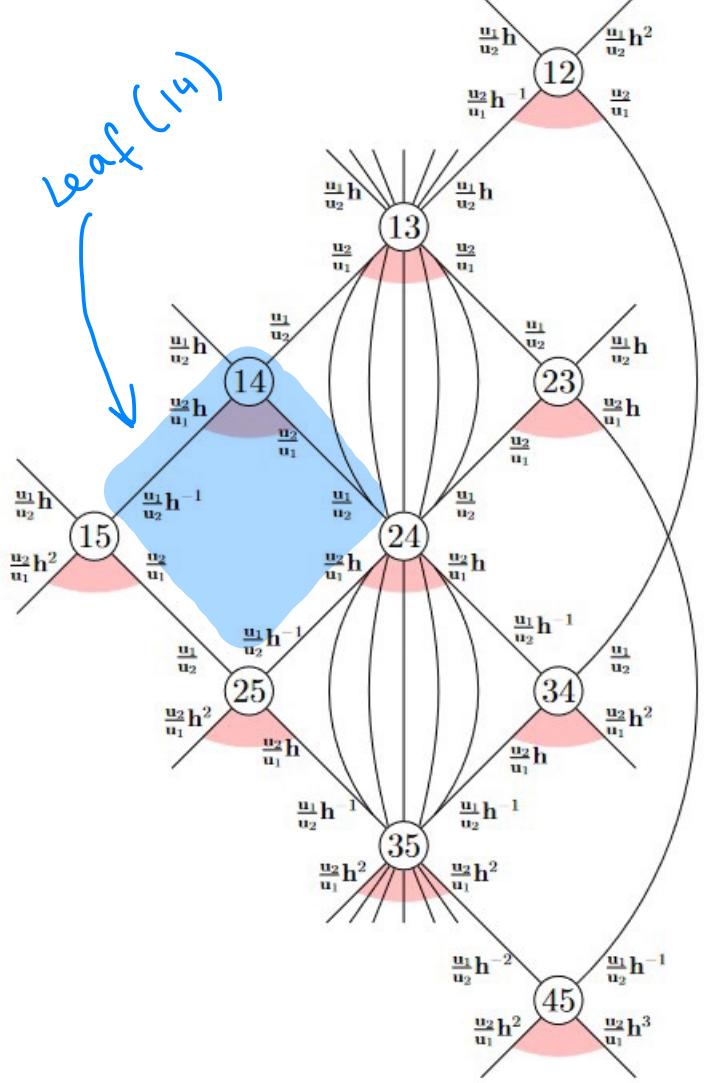


Towards

$$\text{Stab}_p \in H_T^*(N(Q))$$

↑  
(torus fixed point)

- fix  $\mathbb{C}^* \xrightarrow{\delta} T$   
 $z \mapsto (z, z^2, z^3, \dots, z^n, 1)$
- $p \in N(Q)^T$      $\text{Leaf}(p) = \{x \in N(Q) : \lim_{z \rightarrow 0} \delta(z)x = p\}$
- $p' \leq p$       if       $\overline{\text{Leaf}(p)} \ni p'$
- $\text{slope}(p) := \bigcup_{p' \leq p} \text{Leaf}(p')$



def  
[MO]  $\text{Stab}_p \in H_+^*(N(Q))$  is the unique class

- support axiom:

supported on  $\text{Slope}(p)$

- normalization axiom:

$$\text{Stab}_p|_p = e(\nu(\text{Slope}_p))$$

- boundary axiom:

$\text{Stab}_p|_q$  divisible by  $t$  for  $p \neq q$

Stab<sub>14</sub>

@ 12, 13, 23 = 0

@ 14 =  $(z_1 - z_2)(z_1 - z_2 + h)$

@ 15 divisible by  $(z_1 - z_2 + h)$

divisible by  $h$

@ 24 divisible by  $(z_1 - z_2)$

divisible by  $h$

@ 25 divisible by  $h$

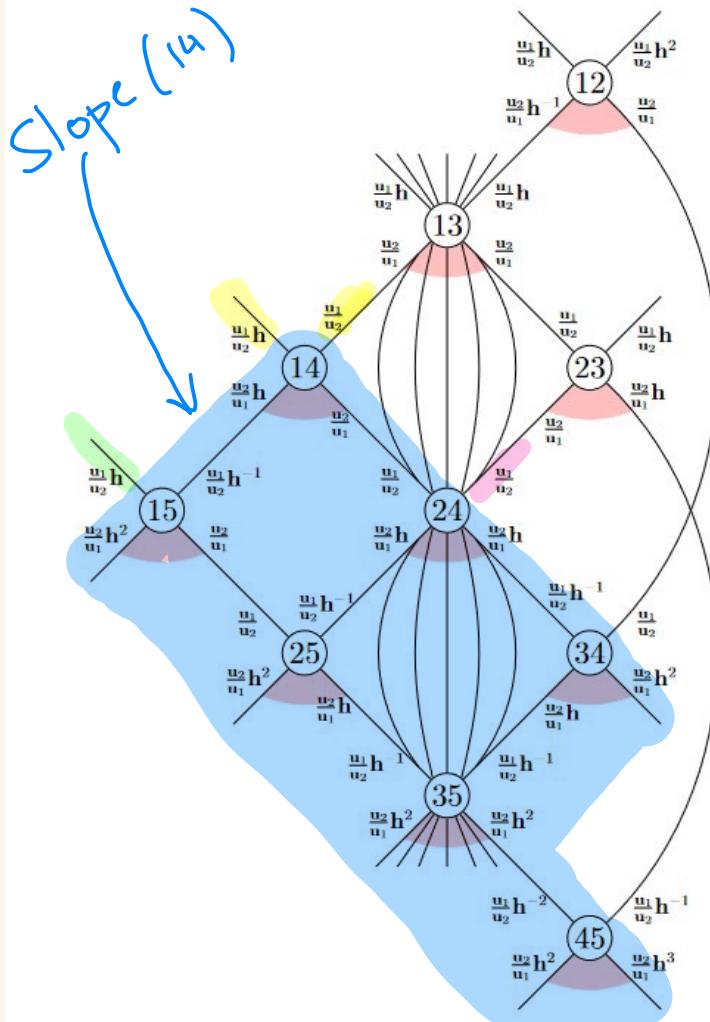
@ 34 divisible by  $(z_1 - z_2)$

divisible by  $h$

@ 35 divisible by  $h$

@ 45 divisible by  $(z_1 - z_2 - h)$

divisible by  $h$



Stab's define geometric R-matrices & quantum group actions.  
[MO]

- $\zeta = (z, z^2, z^3, \dots, z^n, 1) \quad H_T^*(N(Q)^T) \xrightarrow{\text{Stab}_\zeta} H^*(N(Q))$

$$1_p \xrightarrow{\hspace{1cm}} \text{Stab}_\zeta p$$

- other 1-parameter subgroups also define Stab's

$$\begin{array}{ccc} H_T^*(N(Q)^T) & \xrightarrow{\text{Stab}_\zeta} & H^*(N(Q)) \\ \vdots & & \\ \xrightarrow{\text{Stab}_{\zeta'}} & & \end{array}$$

- $\text{Stab}_\zeta^{-1} \circ \text{Stab}_{\zeta'} =: \text{"geometric R-matrix"}$

$$\mathcal{N} := T^* \mathrm{Gr}_0 \mathbb{C}^2 \sqcup T^* \mathrm{Gr}_1 \mathbb{C}^2 \sqcup T^* \mathrm{Gr}_2 \mathbb{C}^2$$

$$H_T^*(\mathcal{N}^\top) \xrightarrow[\text{Stab}_{g'}]{\text{Stab}_g} H^*(\mathcal{N})$$

$$g = (z_1, z^2, 1)$$

$$g' = (z^2, z_1, 1)$$

$$T^* \mathrm{Gr}_0 \mathbb{C}^2$$

$$| \mapsto$$

$$|$$

$$T^* \mathrm{Gr}_1 \mathbb{C}^2$$

$$|_{10} \mapsto$$

$$(z_2 - z_1, 0)$$

$$|$$

$$(z_2 - z_1 + \hbar, \hbar)$$

$$|_{01} \mapsto$$

$$(\hbar, z_1 - z_2 + \hbar)$$

$$(0, z_1 - z_2)$$

$$T^* \mathrm{Gr}_2 \mathbb{C}^2$$

$$| \mapsto$$

$$|$$

$$\text{Stab}_2^{-1} \circ \text{Stab}_{g'} =$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{z_1 - z_2}{z_1 - z_2 + \hbar} & \frac{\hbar}{z_1 - z_2 + \hbar} & 0 \\ 0 & \frac{\hbar}{z_1 - z_2 + \hbar} & \frac{z_1 - z_2}{z_1 - z_2 + \hbar} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

So far :

$$\text{Stab}_p \in H_T^*(N(Q))$$

↑  
T-fixed point of  $N(Q)$

- defined axiomatically
- remark : main ingredients of defining quantum group actions on  $H_T^*(N(Q))$ .

# THE COINCIDENCE !!!

$$T^* \text{Gr}_2 \mathbb{C}^4$$

$$\mathcal{N}\left(\begin{array}{ccc|c} 1 & 2 & 1 \\ \bullet & \bullet & \bullet & \\ \hline & & & \square_2 \end{array}\right)$$

[RSVZ  
2020]

intimate relationship  
between their  
Stable Envelopes

dim = 8

# fix pts = 6

$T^4 \times \mathbb{C}_{\hbar}^*$  action

dim = 4

# fix pts = 6

$T^2 \times \mathbb{C}_{\hbar}^*$  action

8

$$T^* \text{Gr}_2 \mathbb{C}^4$$



$$\mathcal{N}\left(\begin{array}{c} 1 \\ 2 \\ 1 \end{array} \middle| \boxed{2} \right)$$

4

[RSV2]

12

$$T^* \text{Gr}_2 \mathbb{C}^5$$



$$\mathcal{N}\left(\begin{array}{c} 1 \\ 2 \\ 2 \\ 1 \end{array} \middle| \boxed{1}, \boxed{1} \right)$$

4

64

$$T^* \mathcal{F}_{2,6,10}$$



$$\mathcal{N}\left(\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 5 \\ 4 \\ 2 \end{array} \middle| \boxed{2}, \boxed{1} \right)$$

16

8

$$\mathcal{N}\left(\begin{array}{c} 1 \\ 1 \\ 2 \\ 1 \end{array} \middle| \boxed{2}, \boxed{2}, \boxed{2} \right)$$



$$\mathcal{N}\left(\begin{array}{c} 1 \\ 1 \\ 2 \\ 1 \end{array} \middle| \boxed{1}, \boxed{1}, \boxed{2}, \boxed{1} \right)$$

10

$$T^* G/B$$



$$T^* G^L/B^L$$

[RW 2020]

32

$$T^* \mathcal{F}_{2,5,7}$$

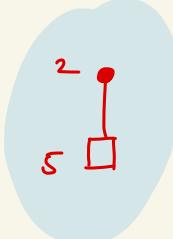
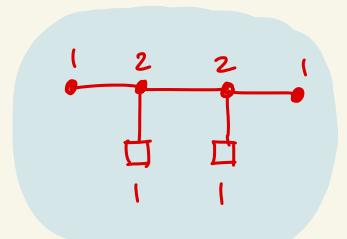


$$\mathcal{N}\left(\begin{array}{c} 3 \\ 2 \\ 1 \end{array} \right)$$

dim

① What exactly is the relationship between  
Stable Envelopes of 3d mirror dual  
spaces ?

② How to find the 3d mirror dual ?

( ie what is the combinatorics that  
connects  with  ? )

( what is the mirror of  $T^* \mathbb{F}_{2,5,7}$  ? )

Cherkis bow varieties  
 $C(\dots)$

type-A Nakajima quiver varieties

$$T^* \mathrm{Gr}_2 \mathbb{C}^4$$

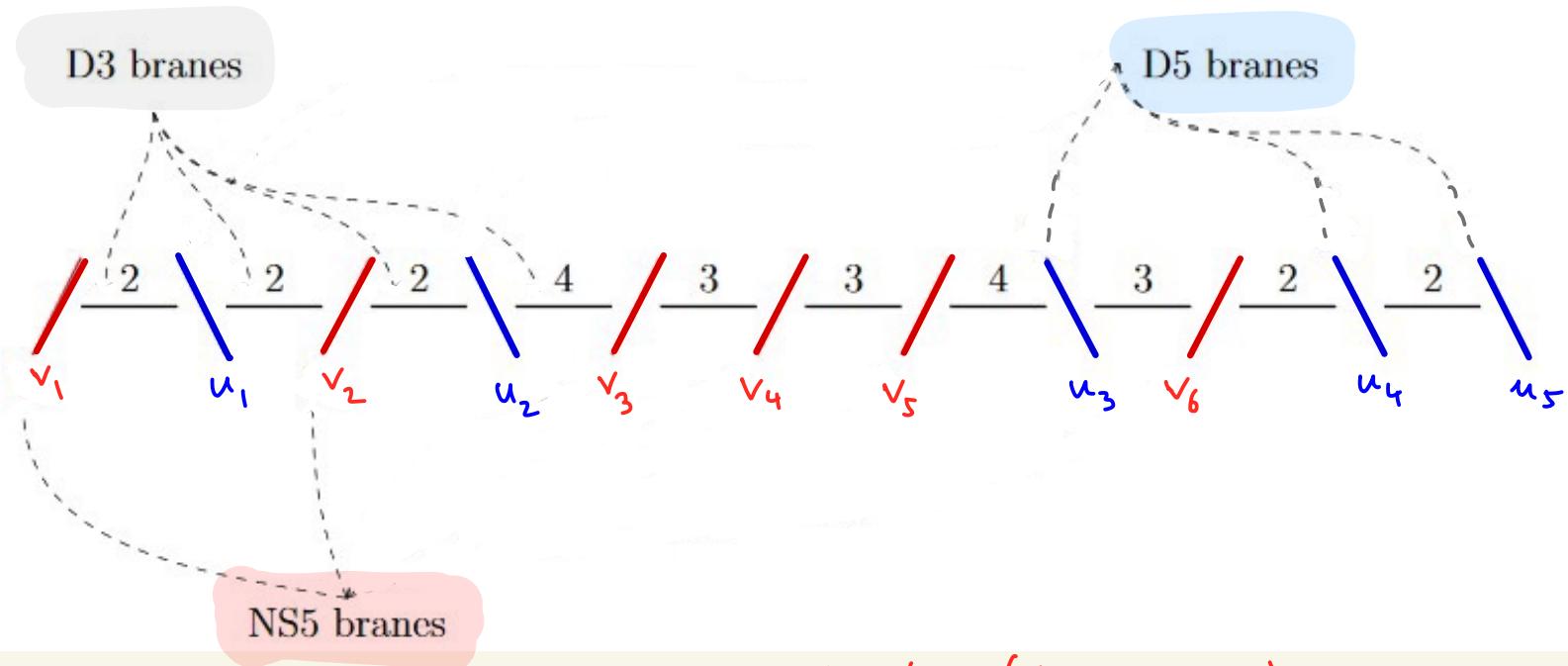
$T^* \mathcal{F}_{2,5,7}$

$T^* \mathcal{T}_{1,2,3,4}$

$$\mathcal{N} \left( \begin{array}{c} 2 & 2 & 1 & 4 \\ \bullet & \bullet & \bullet & \bullet \\ \square & \square & \square & \square \\ 2 & & & 1 \\ | & & & | \end{array} \right)$$

$$\mathcal{N} \left( \begin{array}{c} \cdot \\ \square \\ \cdot \end{array} \middle| \begin{array}{cc} \cdot & \cdot \\ \square & \square \end{array} \right)$$

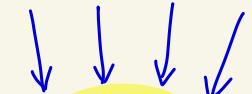
## Brane diagrams



$v_i$ : Kähler (dynamical) variables  
 $u_i$ : equivariant variables

brane  
diagram  
 $\mathcal{D}$

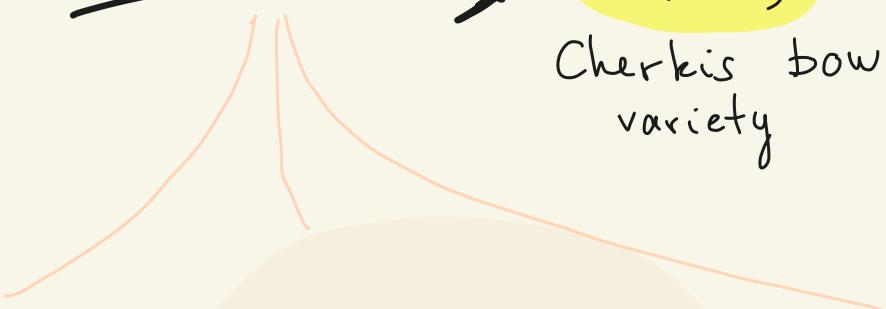
tautological bundles,  
one for each D3 brane



$C(\mathcal{D})$

$T^{D5 \text{ branes}}$

Cherkis bow  
variety



Cherkis:  
moduli space of  
unitary instantons  
on multi-Taub-NUT  
spaces  
(key: Nahm's  
equation)

Nakajima-Takayama  
Hamiltonian reduction  
of representations  
of certain quivers  
with relations

$\sim$

Rozansky = R  
"symplectic  
intersection"  
of generalized  
Lagrange  
varieties

$$\dim(C(D)) = \sum_{U \in D5} \left[ (d_{u_-} + 1)d_{u_-} + (d_{u_+} + 1)d_{u_+} \right]$$

$$+ \sum_{V \in NS5} 2 d_{v^+} d_{v^-} - 2 \sum_{X \in D3} d_X^2$$

example

$$\begin{aligned} \dim(C(\text{Diagram})) &= 2 \cdot 1 + 2 \cdot 1 \\ &\quad + 2 \cdot 0 \cdot 1 + 2 \cdot 1 \cdot 0 - 2(1^2 + 1^2 + 1^2 + 1^2) \\ &= 4 \end{aligned}$$

How are  $\mathcal{N}$ (quiver) special cases?



Examples  $T^* \mathbb{P}^1 = \mathcal{N}\left(\begin{smallmatrix} 1 \\ 2 \end{smallmatrix}\right) = \mathcal{C}\left(\begin{smallmatrix} 1 & 1 & 1 \end{smallmatrix}\right)$

$$T^* \text{Gr}_2 \mathbb{C}^4 = \mathcal{N}\left(\begin{smallmatrix} 2 \\ 4 \end{smallmatrix}\right) = \mathcal{C}\left(\begin{smallmatrix} 2 & 2 & 2 & 2 \end{smallmatrix}\right)$$

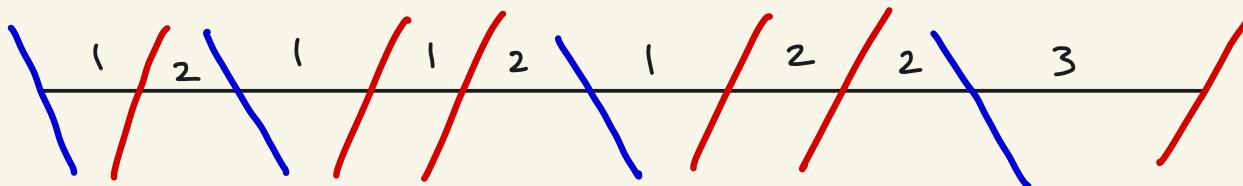
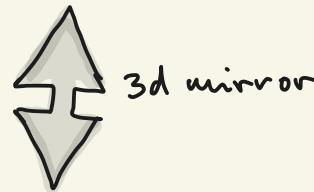
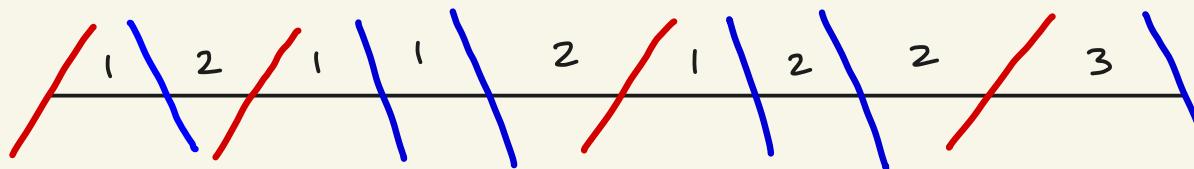
$$T^* \mathcal{F}_{1,2,3,4} = \mathcal{N}\left(\begin{smallmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \end{smallmatrix}\right) = \mathcal{C}\left(\begin{smallmatrix} 1 & 2 & 3 & 3 & 3 & 3 & 3 \end{smallmatrix}\right)$$

$$\mathcal{N}\left(\begin{smallmatrix} 1 & 1 \\ 1 & 1 \end{smallmatrix}\right) = \mathcal{C}\left(\begin{smallmatrix} 1 & 1 & 1 & 1 & 1 \end{smallmatrix}\right)$$

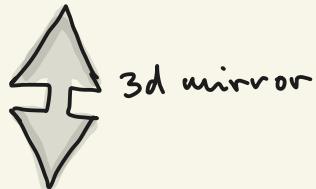
Observe  $\frac{k}{k}$

"cobalanced brane diagram"

3D mirror symmetry for bow varieties:



$$\underline{\text{Ex}} \quad T^* \mathbb{P}^2 = \mathcal{N} \left( \begin{smallmatrix} & 1 \\ 1 & \\ \square & 3 \end{smallmatrix} \right) = C \left( \begin{array}{c|c|c|c|c|c} \textcolor{red}{1} & 1 & 1 & 1 & 1 & \textcolor{red}{1} \\ \hline & & & & & \end{array} \right) \quad \text{dim 4}$$

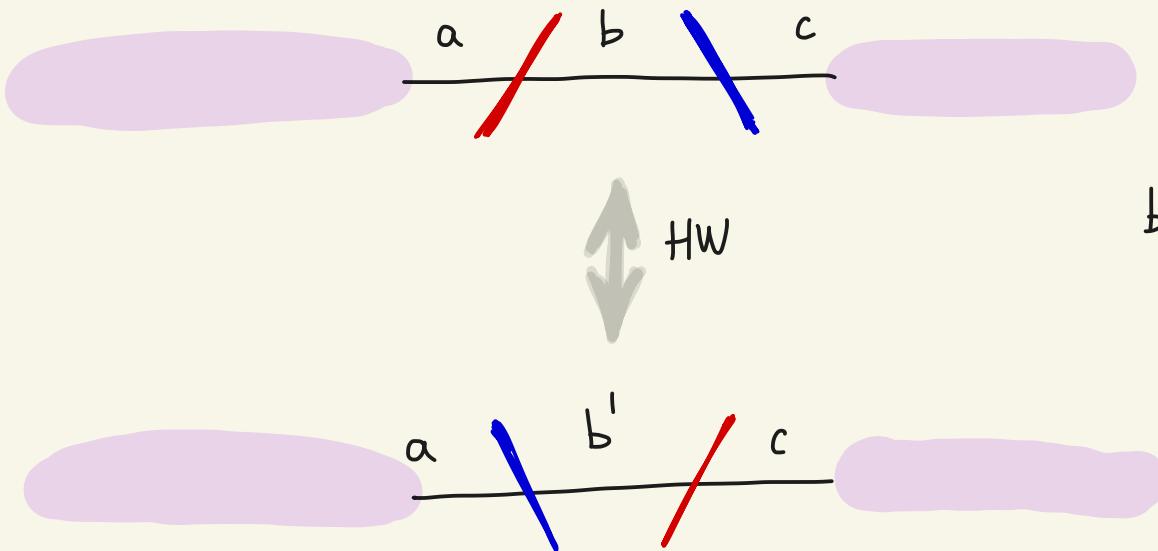


$$C \left( \begin{array}{c|c|c|c|c|c} \textcolor{blue}{1} & \textcolor{red}{1} & \textcolor{red}{1} & \textcolor{red}{1} & \textcolor{red}{1} & \textcolor{blue}{1} \\ \hline & & & & & \end{array} \right) \quad \text{dim 2}$$

not cobalanced, ie not  $\mathcal{N}(\dots)$

... but ... <to be continued>

Hanany - Witten transition on brane diagrams.



$$b + b' = a + c + 1$$

(why? later:  
"brane charge")

Thm  $C(\mathcal{D}) \approx C(HW(\mathcal{D}))$

$$\underline{\text{Ex}} \quad T^*\mathbb{P}^2 = \mathcal{N}\left(\begin{array}{c|c|c} 1 & & \\ \hline & 1 & \\ \hline & & 3 \end{array}\right) = C\left(\begin{array}{ccccccccc} 1 & / & 1 & | & 1 & / & 1 & | & 1 \end{array}\right)$$

3d mirror

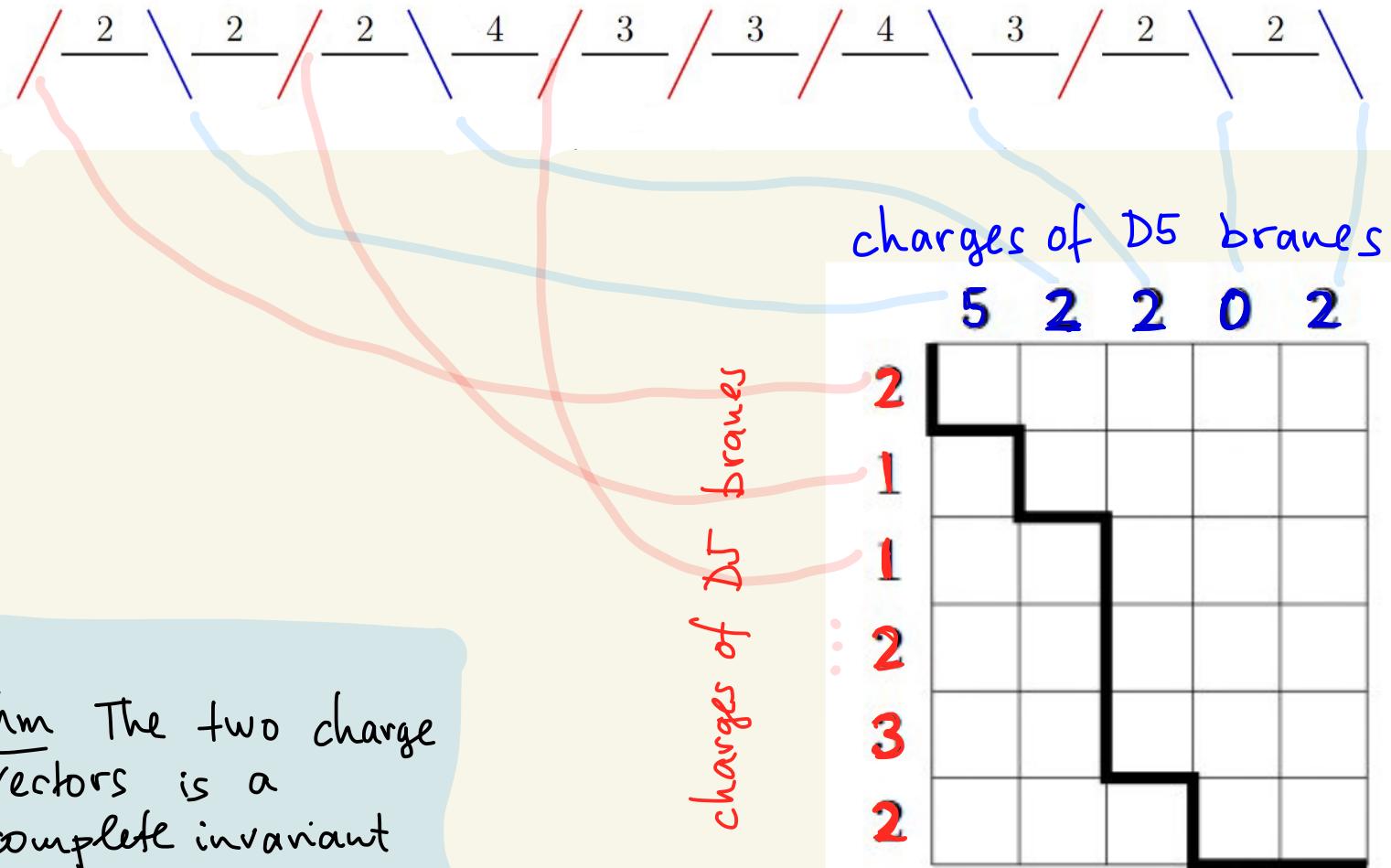
$$\begin{aligned}
 & \text{HW} \rightarrow C\left(\begin{array}{c|c|c|c|c|c} \diagup & \diagdown & \diagup & \diagdown & \diagup & \diagdown \\ \diagdown & \diagup & \diagdown & \diagup & \diagdown & \diagup \end{array}\right) \\
 \text{HW} \rightarrow & C\left(\begin{array}{c|c|c|c|c|c} \diagup & \diagdown & \diagup & \diagdown & \diagup & \diagdown \\ \diagdown & \diagup & \diagdown & \diagup & \diagdown & \diagup \end{array}\right) = C\left(\begin{array}{c|c|c|c|c|c} \diagup & \diagdown & \diagup & \diagdown & \diagup & \diagdown \\ \diagdown & \diagup & \diagdown & \diagup & \diagdown & \diagup \end{array}\right) \\
 & \stackrel{''}{=} N\left(\begin{array}{c|c} \diagup & \diagdown \\ \diagdown & \diagup \end{array}\right)
 \end{aligned}$$

$$\Rightarrow T^* \mathbb{P}^2 \quad \xleftarrow{\text{3d mirror}} \quad \mathcal{N}\left(\begin{smallmatrix} 1 & 1 \\ 1 & 1 \end{smallmatrix}\right)$$

def brane charge

$$\text{charge} \left( \begin{array}{c} \text{NS5 brane} \\ \hline k \cancel{/} l \end{array} \right) := l - k + \#\{\text{D5-branes left of it}\}$$

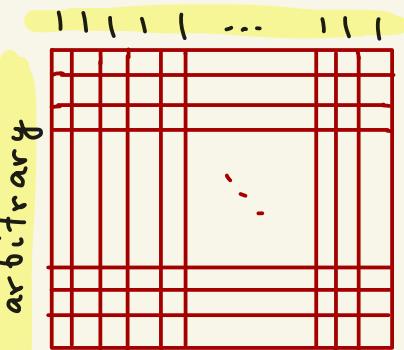
$$\text{charge} \left( \begin{array}{c} \text{D5 brane} \\ \hline k \cancel{/} l \end{array} \right) := k - l + \#\{\text{NS5-branes right of it}\}$$



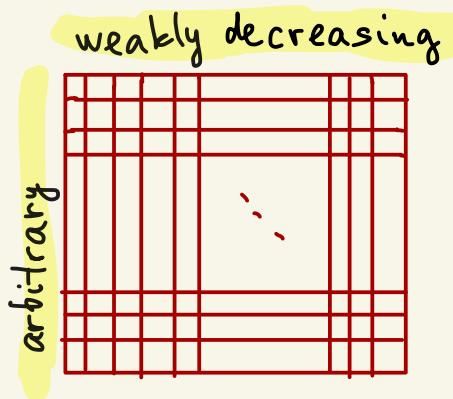
Thm The two charge vectors is a complete invariant of HW class

Thm (up to HW transitions)

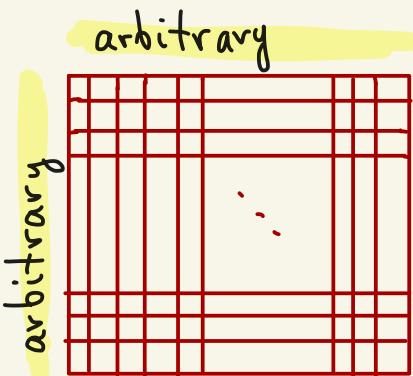
$T^*G/P$



$N(\text{quiver})$



$C(\text{brane diagram})$

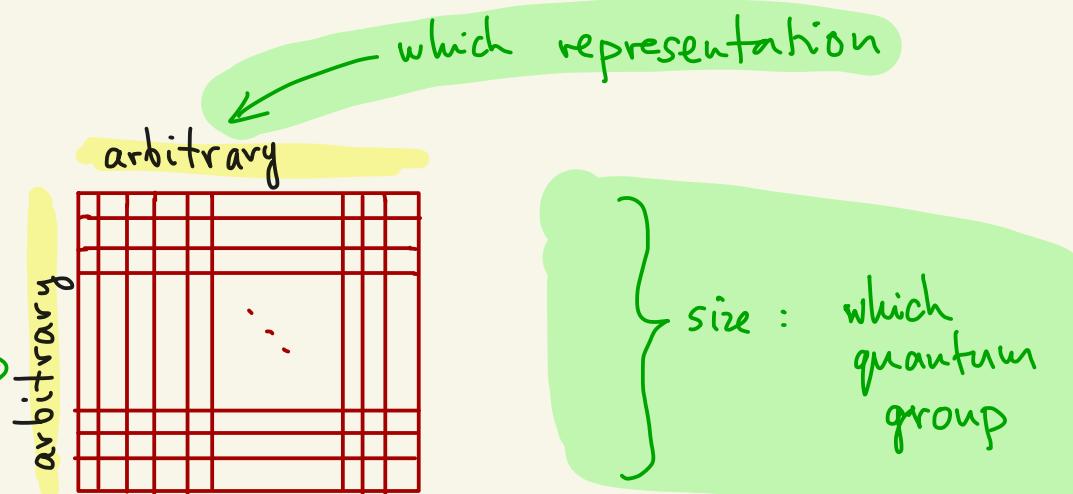


closed for transpose !

Digression : Okounkov's theory : geometric construction of quantum group actions on  $H_T^*, K_T, EU_T$ .

Expectation :  
(known for quivers)

which weight space of the representation

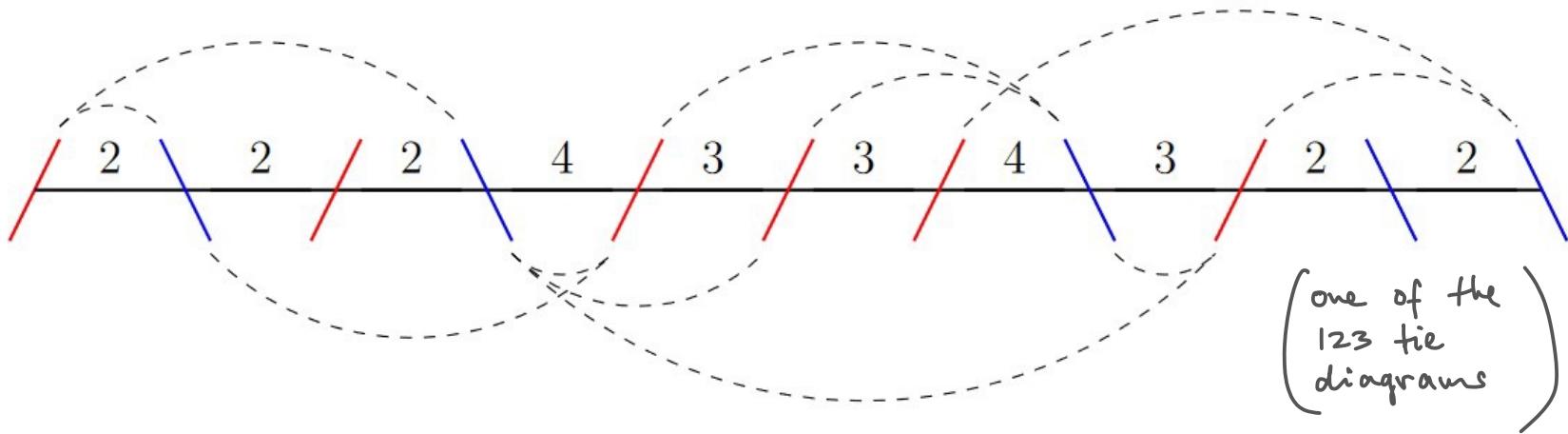


The first data we need for

- $H_T^*(C(D))$
- stable envelopes:

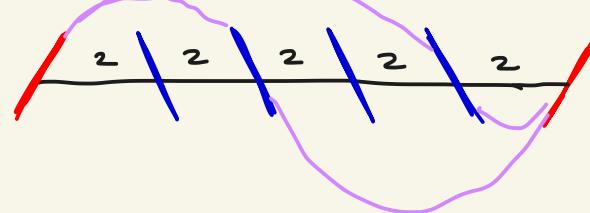
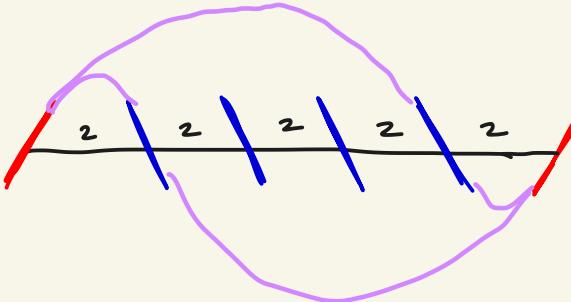
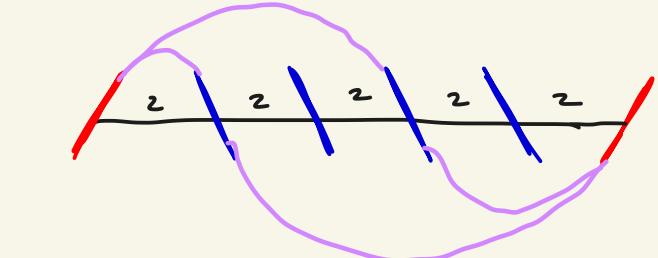
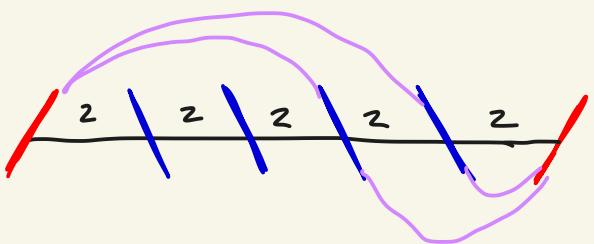
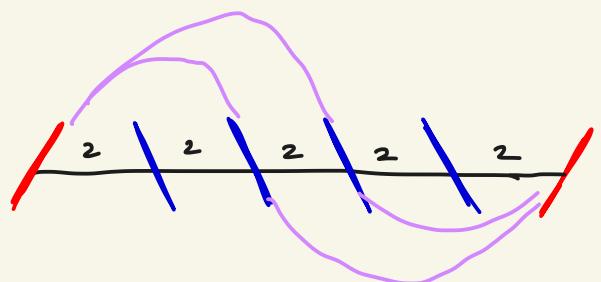
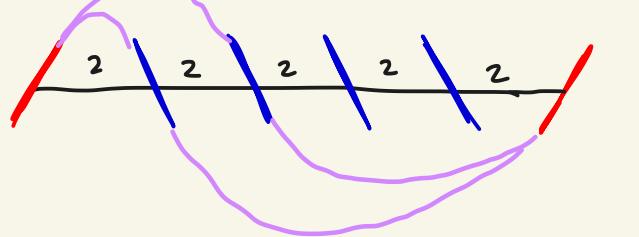
torus fixed points

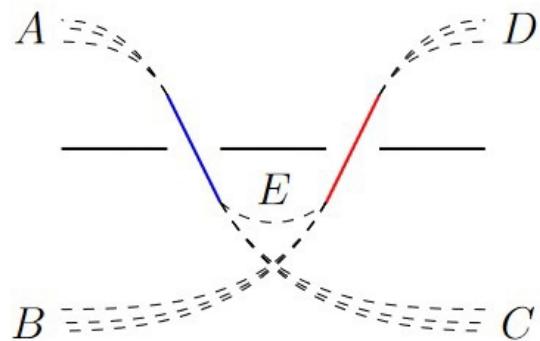
fixed points  $\overset{1:1}{\leftrightarrow}$  tie diagrams



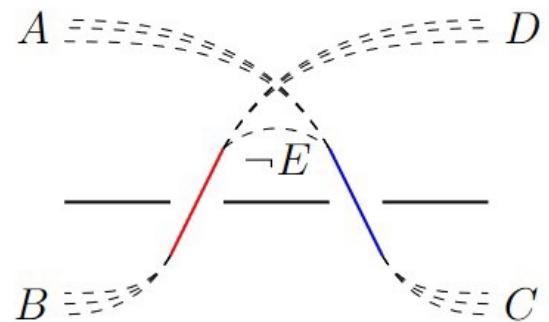
- a tie must connect 5-branes of different kinds
- each D3 brane to be covered as many times as its multiplicity

fixed points of  $T^* \text{Gr}_2 \mathbb{C}^4$ :

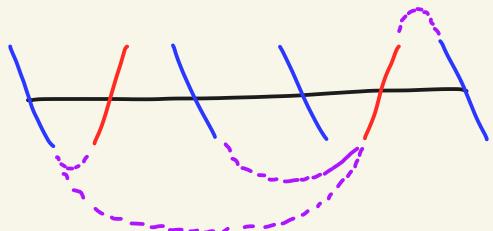




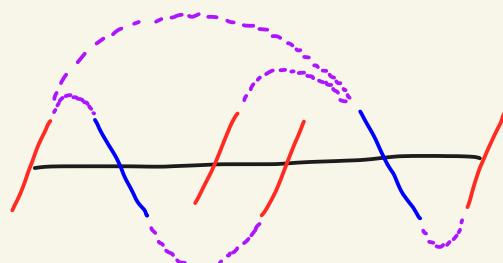
HW transition  
on fixpoints



R-III



3d mirror  
on fixedpoints  
horizontal  
reflection



$$\begin{array}{c} / \quad 2 \quad \backslash \quad 2 \quad / \quad 2 \quad \backslash \quad 4 \quad / \quad 3 \quad / \quad 3 \quad / \quad 4 \quad \backslash \quad 3 \quad / \quad 2 \quad \backslash \quad 2 \quad \backslash \\ \text{---} \quad \text{---} \end{array}$$

binary contingency tables

BCT : 0-1-matrix  
with row &  
column sums  
the charge vectors

Thm

fix pts  $\longleftrightarrow$  BCT's

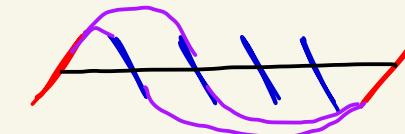
one of the 123 BCTs

	5	2	2	0	2
2	1	1	0	0	0
1	1	0	0	0	0
1	0	0	1	0	0
2	1	0	1	0	0
3	1	1	0	0	1
2	1	0	0	0	1

$\text{Gr}_2 \mathbb{C}^4$

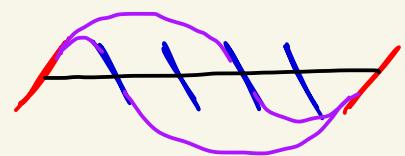
0

$\{1,2\}$



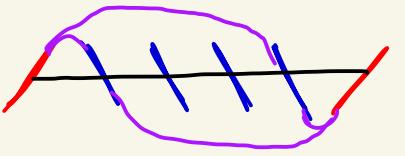
□

$\{1,3\}$



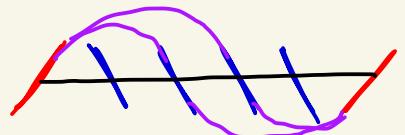
田

$\{1,4\}$



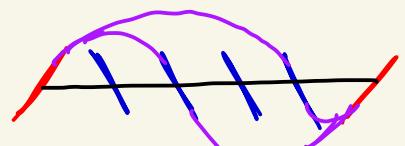
日

$\{2,3\}$



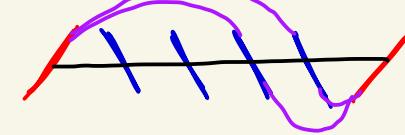
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$\{2,4\}$



田

$\{3,4\}$



1	1	1	1	1
2	1	1	0	0
2	0	0	1	1

1	1	1	1	1
2	1	0	1	0
2	0	1	0	1

1	1	1	1	1
2	1	0	0	1
2	0	1	1	0

1	1	1	1	1
2	0	1	1	0
2	1	0	0	1

1	1	1	1	1
2	0	1	0	1
2	1	0	1	0

1	1	1	1	1
2	0	0	1	1
2	1	1	0	0

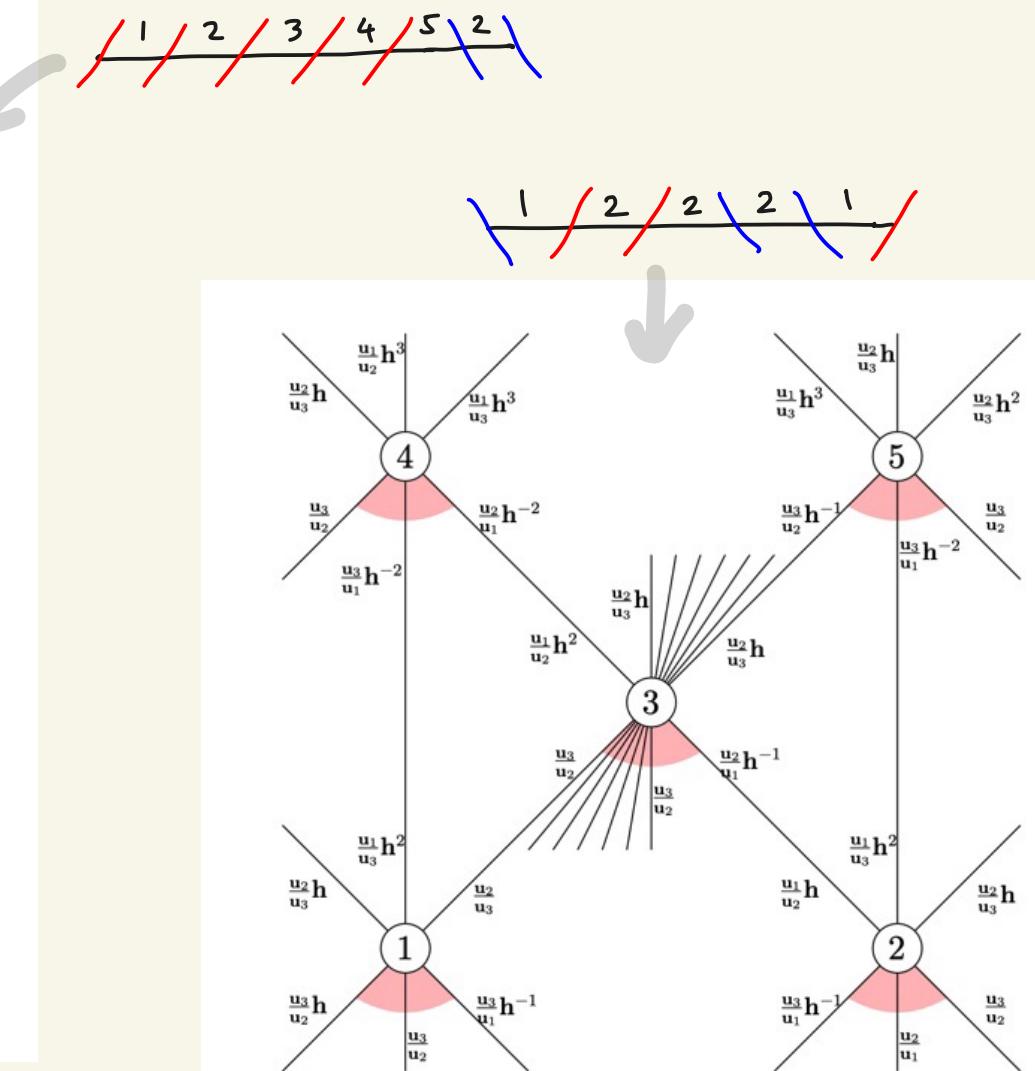
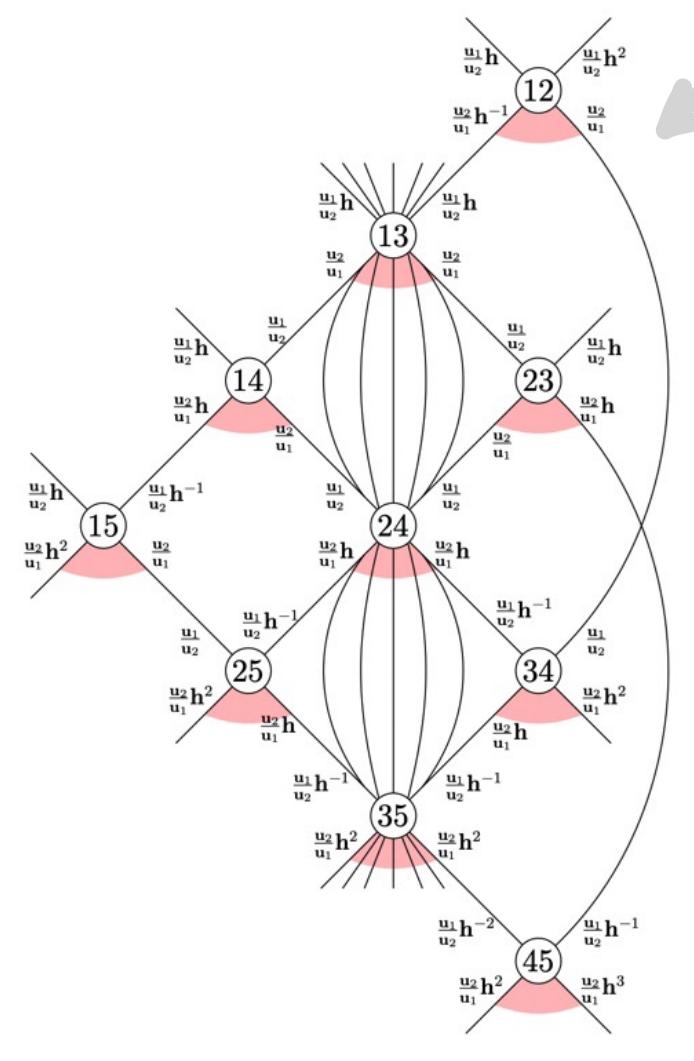
fixed points  
invariant curves  
(with weight)

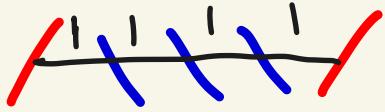


moment  
graph



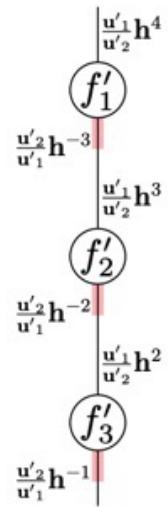
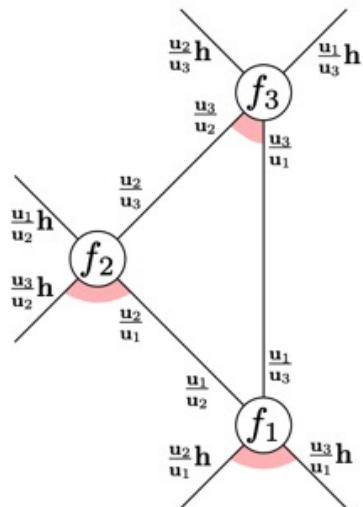
$\text{Stab}_P$



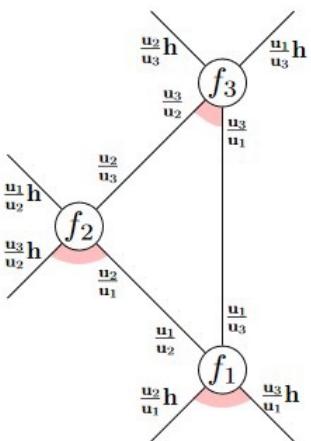


$T^* \mathbb{P}^2$

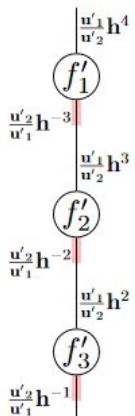
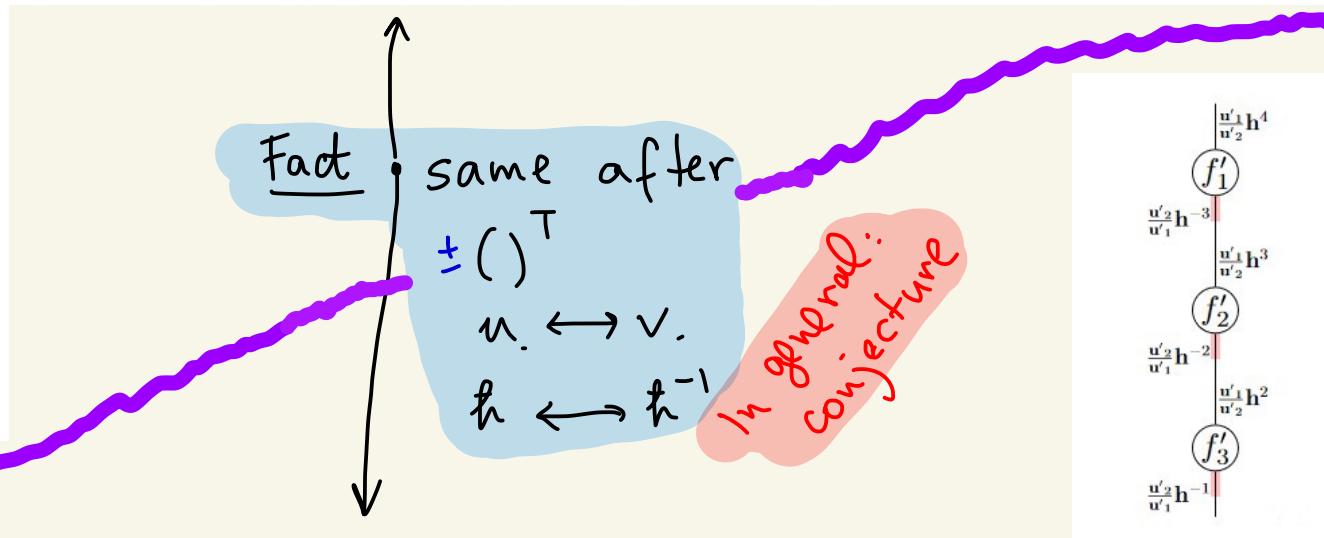
3d mirror  $(T^* \mathbb{P}^2)$



$$\begin{aligned} T^*\mathbb{P}^2 &= \mathcal{N}\left(\begin{array}{c|c} \square & \square \\ \square & 3 \end{array}\right) \\ &= \mathcal{C}\left(\begin{array}{ccccccc} \diagup & \diagdown & \diagup & \diagdown & \diagup & \diagdown & \diagup \end{array}\right) \end{aligned}$$



	$f_1$	$f_2$	$f_3$
$f_1$	$\theta\left(\frac{u_1}{u_2}\right)\theta\left(\frac{u_1}{u_3}\right)\theta\left(\frac{v_2}{v_1}h^4\right)$	0	0
$f_2$	$\theta(h)\theta\left(\frac{u_1}{u_3}\right)\theta\left(\frac{u_2 v_2}{u_1 v_1}h^3\right)$	$\theta\left(\frac{u_1}{u_2}h\right)\theta\left(\frac{u_2}{u_3}\right)\theta\left(\frac{v_2}{v_1}h^3\right)$	0
$f_3$	$\theta(h)\theta\left(\frac{u_2}{u_1}h\right)\theta\left(\frac{u_3 v_2}{u_1 v_1}h^2\right)$	$\theta(h)\theta\left(\frac{u_1}{u_2}h\right)\theta\left(\frac{u_3 v_2}{u_2 v_1}h^2\right)$	$\theta\left(\frac{u_2}{u_3}h\right)\theta\left(\frac{u_1}{u_3}h\right)\theta\left(\frac{v_2}{v_1}h^2\right)$



	$f'_1$	$f'_2$	$f'_3$
$f'_1$	$\theta\left(\frac{u'_1}{u'_2}h^4\right)\theta\left(\frac{v'_2}{v'_1}\right)\theta\left(\frac{v'_3}{v'_1}\right)$	$\theta(h)\theta\left(\frac{v'_3}{v'_1}\right)\theta\left(\frac{v'_2 u'_2}{v'_1 u'_1}h^{-3}\right)$	$\theta(h)\theta\left(\frac{v'_2}{v'_1}h^{-1}\right)\theta\left(\frac{v'_3 u'_2}{v'_1 u'_1}h^{-2}\right)$
$f'_2$	0	$\theta\left(\frac{u'_1}{u'_2}h^3\right)\theta\left(\frac{v'_2}{v'_1}h\right)\theta\left(\frac{v'_3}{v'_2}\right)$	$\theta(h)\theta\left(\frac{v'_2}{v'_1}h\right)\theta\left(\frac{v'_3 u'_2}{v'_2 u'_1}h^{-2}\right)$
$f'_3$	0	0	$\theta\left(\frac{u'_1}{u'_2}h^2\right)\theta\left(\frac{v'_3}{v'_2}h\right)\theta\left(\frac{v'_3}{v'_1}h\right)$

$$\begin{aligned} \mathcal{N}\left(\begin{array}{c|c} \square & \square \\ \square & 1 \end{array}\right) &= \\ \mathcal{C}\left(\begin{array}{ccccccc} \diagup & \diagdown & \diagup & \diagdown & \diagup & \diagdown & \diagup \end{array}\right) & \end{aligned}$$

$$\left. \begin{array}{l} x_1 + x_2 + x_3 = 0 \\ y_1 + y_2 + y_3 = 0 \end{array} \right\} \Rightarrow \frac{1}{x_1 x_2} + \frac{1}{x_2 x_3} + \frac{1}{x_3 x_1} = \frac{1}{y_1 y_2} + \frac{1}{y_2 y_3} + \frac{1}{y_3 y_1}$$

H<sub>T</sub>\*

rational limit ( $\sin x \sim x$ )

$$x_1 + x_2 + x_3 = 0, \quad y_1 + y_2 + y_3 = 0 \Rightarrow$$

$$\cot(x_1) \cot(x_2) + \cot(x_2) \cot(x_3) + \cot(x_3) \cot(x_1) = \cot(y_1) \cot(y_2) + \cot(y_2) \cot(y_3) + \cot(y_3) \cot(y_1)$$

K<sub>T</sub>

↑ trigonometric limit ( $q \rightarrow 1$ )

$$\left. \begin{array}{l} x_1 x_2 x_3 = 1 \\ y_1 y_2 y_3 = 1 \end{array} \right\} \Rightarrow \delta(x_1, y_2) \delta(x_2, \frac{1}{y_1}) + \delta(x_2, y_3) \delta(x_3, \frac{1}{y_2}) + \delta(x_3, y_1) \delta(x_1, \frac{1}{y_3}) = 0$$

Ell<sub>T</sub>

Thank you !