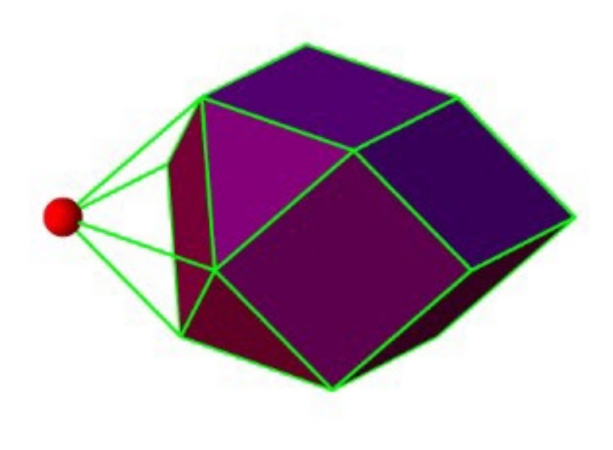
Motivic Chern Classes



$c^{sm} \in H_T^*$

$mC \in K_T$

$Ell \in Ell_T$

$$c^{sm}(\Omega_I) \in H_T^*(Gr_k\mathbb{C}^n) \xrightarrow{Loc} \bigoplus_I \underbrace{H_T^*(x_I)}_{\mathbb{Z}[z_1 \dots z_n]}$$

$$\underbrace{mC}_{\text{"motivic Chern class"}}(\Omega_I) \in K_T(Gr_k\mathbb{C}^n) \xrightarrow{Loc} \bigoplus_I \underbrace{K_T(x_I)}_{\mathbb{Z}[z_1^{\pm 1}, \dots, z_n^{\pm 1}]}$$

im(Loc): $(i-j)$-neighboring components satisfy $z_i - z_j \mid f_I - f_J$

fact <u>same</u> description in K_T

$$z_i - z_j \mid f_I - f_J$$

$$\left(1 - \frac{z_j}{z_i} \mid f_I - f_J\right)$$

$H_T^*(\mathbb{P}^1)$

$$[\bar\Omega_1] = (z_2 - z_1, \quad 0)$$
$$[\bar\Omega_2] = (1, \quad 1)$$

Schur poly's

$$c^{sm}(\Omega_1) = (z_2 - z_1, \quad 0)$$
$$c^{sm}(\Omega_2) = (\hbar, \quad z_1 - z_2 + \hbar)$$

$K_T(\mathbb{P}^1)$

$$[\bar\Omega_1]^K = \left(1 - \frac{z_1}{z_2}, \quad 0\right)$$
$$[\bar\Omega_2]^K = (1, \quad 1)$$

Grothendieck poly's

$$mC(\Omega_1) = \left(1 - \frac{z_1}{z_2}, \quad 0\right)$$
$$mC(\Omega_2) = \left((1+\hbar)\frac{z_1}{z_2}, \quad 1 + \frac{z_2}{z_1}\hbar\right)$$

next slide: axiomatic definition

Thm-Def $mC(\Omega_I)$ = unique class in $K_T(Gr_k\mathbb{C}^n)$

- $mC(\Omega_I)|_I = \prod_{\substack{i\in I\\ j\in\bar{I}\\ i<j}} \left(1-\frac{z_i}{z_j}\right) \cdot \underbrace{\prod_{\substack{i\in I\\ j\in\bar{I}\\ i>j}} \left(1+\hbar\frac{z_i}{z_j}\right)}_{c_I}$

- $mC(\Omega_I)|_J$ divisible by c_J

divisibility by $\hbar$

- $\underbrace{N}_{\text{Newton polygon}}\left(mC(\Omega_I)|_J\right) \subset \underbrace{N}_{\text{Newton polygon}}\left(mC(\Omega_J)|_J\right) - 0$ for $I \neq J$

(• $mC(\Omega_I)|_J = 0$ if $J \not\leq I$)

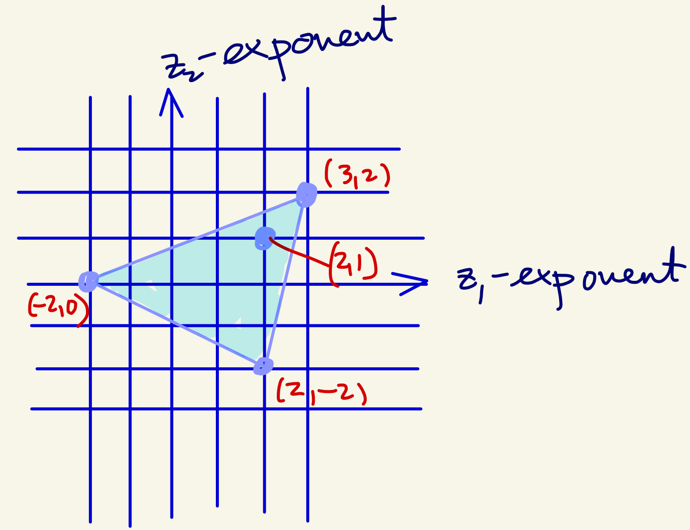
$$f \in \mathbb{Z}[z_1^{\pm 1}, z_2^{\pm 1}, \dots, z_n^{\pm 1}]$$

$$f = \sum_{K \in \mathbb{Z}^n} c_I \cdot \underline{z}^K$$

$$N(f) := \text{convex hull of } \left(K \ : \ c_K \neq 0 \right)$$

<u>ex</u>

$$N\left(\underset{\substack{\uparrow \\ (3,2)}}{z_1^3 z_2^2} - 7 \underset{\substack{\uparrow \\ (2,-2)}}{\frac{z_1^2}{z_2^2}} + 3 \underset{\substack{\uparrow \\ (2,1)}}{z_1^2 z_2} - 8 \underset{\substack{\uparrow \\ (-2,0)}}{\frac{1}{z_1^2}} \right) =$$

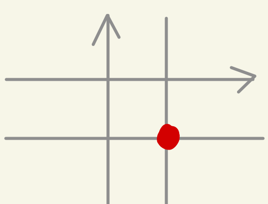


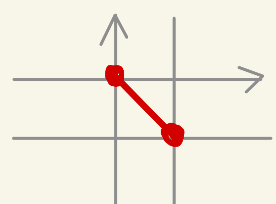
$$mC(\Omega_1) = \left(1 - \frac{z_1}{z_2},\ 0\right)$$

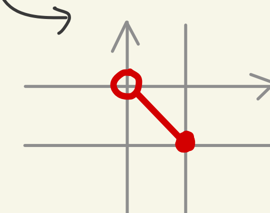
$$mC(\Omega_2) = \left((1+\hbar)\frac{z_1}{z_2},\ 1 + \frac{z_2}{z_1}\hbar\right)$$

Newton polygon axiom here:

$$N\left((1+\hbar)\frac{z_1}{z_2}\right) \subset N\left(1 - \frac{z_1}{z_2}\right) - 0$$







Where are we so far?

- axiomatic definition of $mC(\Omega_I) \in K_T(Gr)$

Facts

✓ • R-matrix property ("trigonometric solution of Yang-Baxter")

✓ • formulas of the type $\sum \prod \prod(\text{rational})$ exist ("trigonometric weight functions")

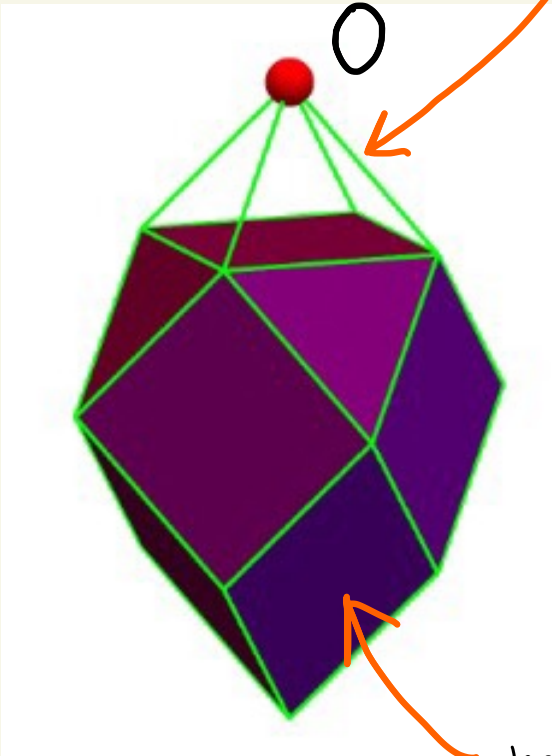
✓ • cotangent interpretation

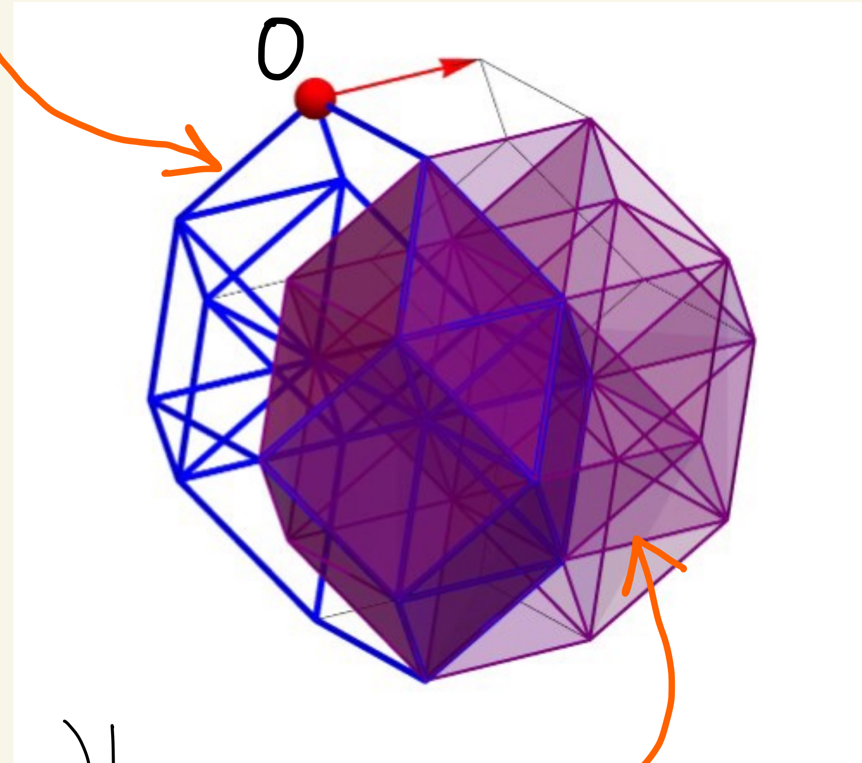
✓ • Bott-Samelson & R-matrix recursions

✓ • MacPherson property ("motivic" class)

$mC(\Omega_J)|_J$

$Gr_2\,\mathbb{C}^4$





$mC(\Omega_I)|_J$