Why $\boxed{\text{"cotangent"}}$ Schubert Calculus

$\mathbb{P}^1$

$$c^{sm}(\Omega_1) = (z_2 - z_1, \quad 0)$$

= $[\{x_1\}]$ fundamental class

$$c^{sm}(\Omega_2) = (\hbar, \quad z_1 - z_2 + \hbar)$$

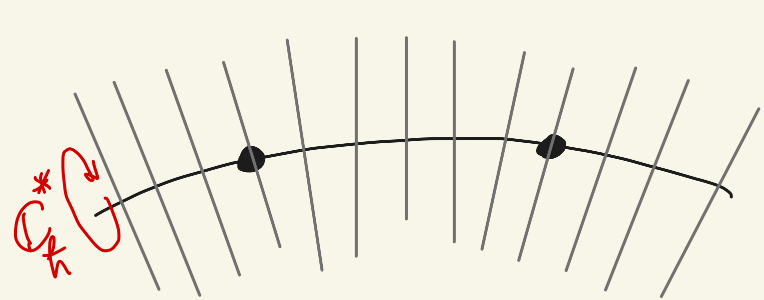
$\neq$ fundamental class

⋮

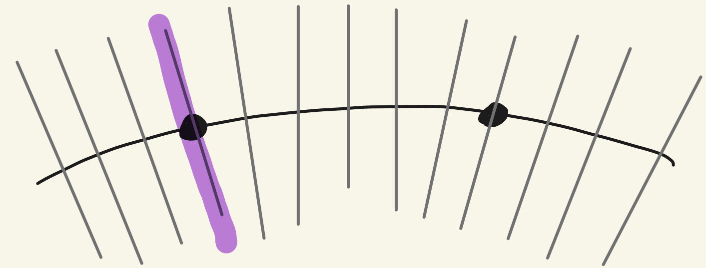
<u>but</u>

⋮

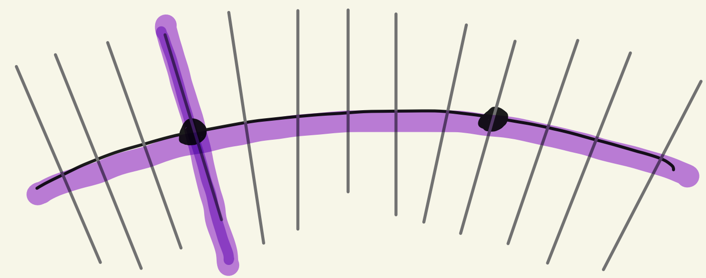
- replace $\mathbb{P}^1$ with $T^*\mathbb{P}^1$ & act by $\mathbb{C}^*_\hbar$ in fibers

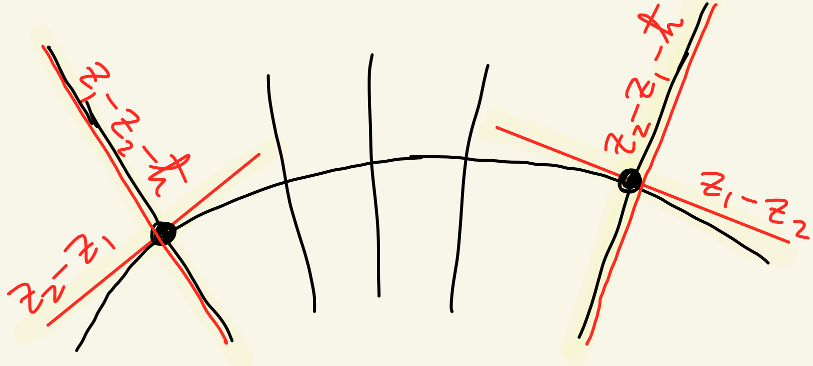


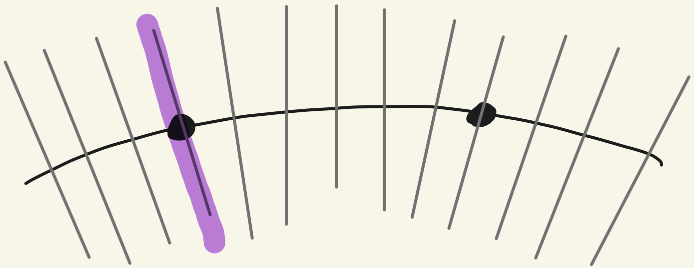
- consider these cycles

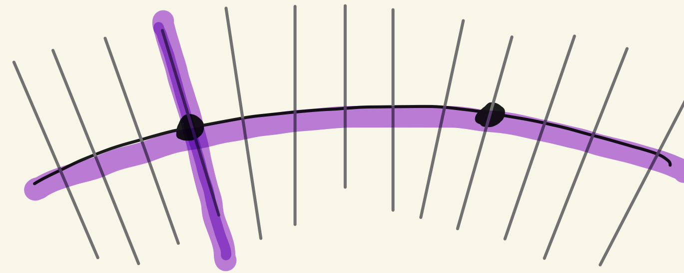


calculate their fundamental classes

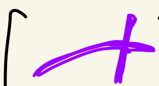








[] $= \left(z_2 - z_1,\ 0\right)$

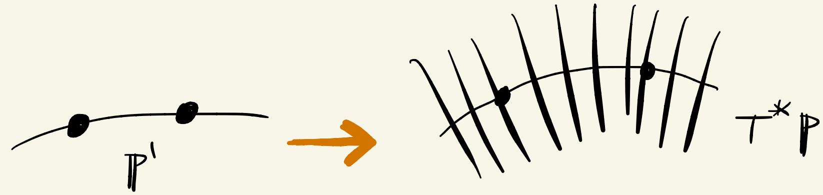
[] $= \left(\begin{matrix} (z_2 - z_1) + \\ (z_1 - z_2 - \hbar) \end{matrix},\ z_2 - z_1 - \hbar \right)$

$= -\left(\hbar,\ z_1 - z_2 + \hbar \right)$

Summary

$X \longrightarrow T^*X$

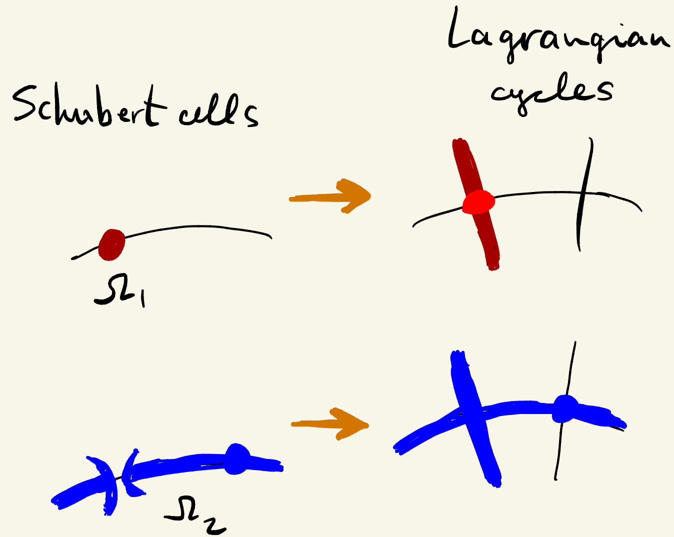
with extra $\mathbb{C}^*_\hbar$ action



$\Omega \longrightarrow$ "associated Lagrangian cycle $\subset T^*X$"

$\Omega_w \to \coprod_{\eta \leq w} c_{w\eta} \cdot C\Omega_\eta$

conormal bundle of Schubert cell



$\Longrightarrow$ $c^{sm} \longrightarrow$ fundamental class