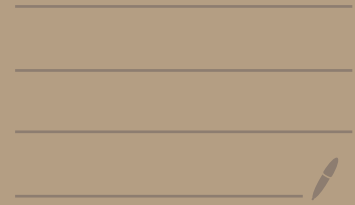


MacPherson property  
of CSM classes

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	12	13	14	23	24	34
$c^{sm}(\Omega_{12})$	$\frac{(z_3 - z_1)(z_4 - z_1)}{(z_3 - z_2)(z_4 - z_2)}$	0	0	0	0	0
$c^{sm}(\Omega_{13})$	$h \dots$	$\frac{(z_2 - z_1)(z_4 - z_1)(z_4 - z_3)}{(z_2 - z_3 + h)}$	0	0	0	0
$c^{sm}(\Omega_{14})$	$h \dots$	$h(z_2 - z_3 + h) \dots$	$\frac{(z_2 - z_1)(z_3 - z_1)}{(z_2 - z_4 + h)(z_3 - z_4 + h)}$	0	0	0
$c^{sm}(\Omega_{23})$	$h \dots$	$h(z_2 - z_3 + h) \dots$	0	$\frac{(z_4 - z_2)(z_4 - z_3)}{(z_1 - z_2 + h)(z_1 - z_3 + h)}$	0	0
$c^{sm}(\Omega_{24})$	$h \dots$	$h(z_2 - z_3 + h) \dots$	$h(z_2 - z_4 + h)(z_3 - z_4 + h) \dots$	$h(-)(-)$	$\frac{(z_3 - z_2)(z_1 - z_2 + h)}{(z_1 - z_4 + h)(z_3 - z_4 + h)}$	0
$c^{sm}(\Omega_{34})$	$h \dots$	$h(z_2 - z_3 + h) \dots$	$h(z_2 - z_4 + h)(z_3 - z_4 + h) \dots$	$h(-)(-)$	$h(-)(-)(-)$	$\frac{(z_1 - z_3 + h)(z_1 - z_4 + h)}{(z_2 - z_3 + h)(z_2 - z_4 + h)}$

$\Sigma$  add together all

$$\sum_I c^{sm}(\Omega_I) = \left( \begin{array}{c|c|c|c} 12 & 13 & 14 & 23 & 24 & 34 \\ \hline (z_3 - z_1 + h)(z_3 - z_2 + h) & & (z_2 - z_1 + h)(z_3 - z_1 + h) & & (z_1 - z_2 + h)(z_3 - z_2 + h) & \\ (z_4 - z_1 + h)(z_4 - z_2 + h) & & (z_2 - z_4 + h)(z_3 - z_4 + h) & & (z_1 - z_4 + h)(z_3 - z_4 + h) & \\ \hline & (z_2 - z_1 + h)(z_4 - z_1 + h) & & (z_1 - z_2 + h)(z_4 - z_2 + h) & & (z_1 - z_3 + h)(z_2 - z_3 + h) \\ & (z_2 - z_3 + h)(z_4 - z_3 + h) & & (z_1 - z_3 + h)(z_4 - z_3 + h) & & (z_1 - z_4 + h)(z_2 - z_4 + h) \end{array} \right)$$

$\Rightarrow$

at each fixed point  $I$  it is  $\prod_{\substack{i \in I \\ j \in \bar{I}}} (z_j - z_i + h)$

but  $c(TGr_2 \mathbb{C}^4)|_I = \prod_{\substack{i \in I \\ j \in \bar{I}}} (1 + z_j - z_i)$

$\Rightarrow$

$$\sum c^{sm}(\Omega_I) = c\left(T\left(\overbrace{\bigsqcup_I \Omega_I}^{\text{smooth}}\right)\right)$$

$(h=1)$

$$c^{sm}(\Omega_{23}) = \dots$$

$$c^{sm}(\Omega_{24}) = \dots$$

$$c^{sm}(\Omega_{34}) = \dots$$

$$\frac{(z_4 - z_2)(z_4 - z_3)}{(z_1 - z_2 + h)(z_1 - z_3 + h)}$$

$$h(-)(-)$$

$$\frac{(z_3 - z_2)(z_1 - z_2 + h)}{(z_1 - z_4 + h)(z_3 - z_4 + h)}$$

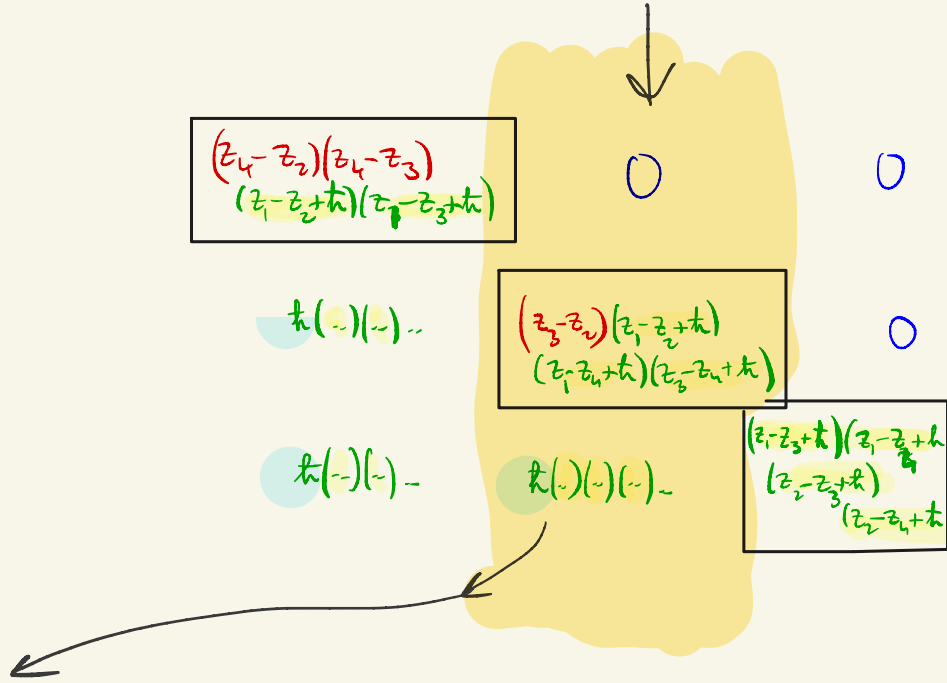
$$h(-)(-)$$

$$h(-)(-)(-)$$

$$\frac{(z_1 - z_3 + h)(z_1 - z_4 + h)}{(z_2 - z_3 + h)(z_2 - z_4 + h)}$$

$$\frac{(z_3 - z_2)(z_1 - z_2 + h)(z_1 - z_4 + h)(z_3 - z_4 + h)}{h(z_1 - z_2 + h)(z_1 - z_4 + h)(z_3 - z_4 + h)}$$

$$\left. \vphantom{\frac{(z_3 - z_2)(z_1 - z_2 + h)(z_1 - z_4 + h)(z_3 - z_4 + h)}{h(z_1 - z_2 + h)(z_1 - z_4 + h)(z_3 - z_4 + h)}} \right\} + = (z_3 - z_2 + h)(z_1 - z_2 + h)(z_1 - z_4 + h)(z_3 - z_4 + h)$$



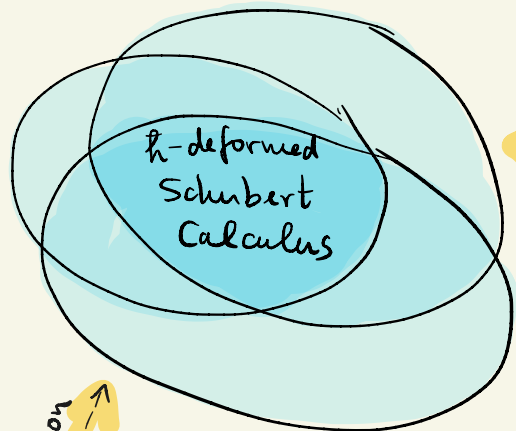
More generally

$$) \text{ if } \bigsqcup_{\mathcal{J}} \Omega_{\mathcal{J}} = M \stackrel{i}{\subseteq} \text{Gr}_k \mathbb{C}^n,$$

$\uparrow$   
smooth cpt

then

$$\sum_{\mathcal{J}} c^{\text{sm}}(\Omega_{\mathcal{J}}) = i_* (c(TM))$$



- overdetermined notion
- generalize to different settings
- different conventions ( $\hbar$  variable or  $\hbar=1$ )
- different names ..

$c(TX)=?$   
if  $X$  is not smooth

Schubert Calc.  
not on  $X$ ,  
but on  $T^*X$

quantum integrable systems  
quantum group representations

general recursions in Schubert Calculus  
Hecke algebras

search for elliptic characteristic classes

KNOWN MATHEMATICS

More generally:  $c^{sm}(f) \in H_T^*(Gr_k \mathbb{C}^n)$  is defined

constructible (T-invariant) function

meaning: can be written as

$$\sum c_V \cdot \mathbb{1}_V$$

indicator function  
of variety  $V$

- additive

$$c^{sm}(f+g) = c^{sm}(f) + c^{sm}(g)$$

$$c^{sm}(\lambda f) = \lambda \cdot c^{sm}(f)$$

- if  $f = \mathbb{1}_M$  compact smooth

then  $c^{sm}(f) = i_* (c(TM))$

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$$c^{sm}(\Omega_I) = c^{sm}(\mathbb{1}_{\Omega_i})$$

Even more generally:

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$C_*^T$

$$: \underbrace{F^T(-)} \rightarrow \underbrace{H_*^T(-)}$$

functor of  
T-invariant  
constructible  
functions on  
 $\mathbb{C}$  algebraic  
varieties

T-equivariant  
homology  
functor

[covariant, pushforward  
defined  
via  $X$ ]

unique natural  
transformation  
of functors  
satisfying

- $\mathbb{1}_M \mapsto c(TM) \cap \mu_M^T$