R- matrix

Basis of $H^*_{T^2}(\mathrm{Gr}_0\mathbb{C}^2 \sqcup \mathrm{Gr}_1\mathbb{C}^2 \sqcup \mathrm{Gr}_2\mathbb{C}^2)$

$c^{sm}(\Omega_\varnothing \subset \mathrm{Gr}_0\mathbb{C}^2) = 1$ $\quad\in H^*_T(\mathrm{Gr}_0\mathbb{C}^2)$

$c^{sm}(\Omega_1 \subset \mathrm{Gr}_1\mathbb{C}^2) = z_2 - t$

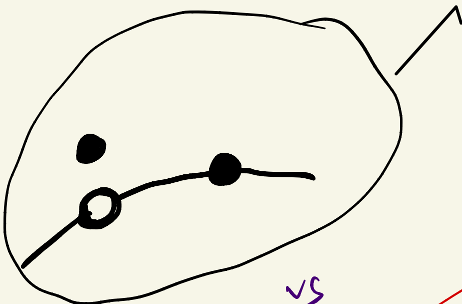
$c^{sm}(\Omega_2 \subset \mathrm{Gr}_1\mathbb{C}^2) = z_1 - t + \hbar$

$\left.\right\} \in H^*_T(\mathrm{Gr}_1\mathbb{C}^2)$

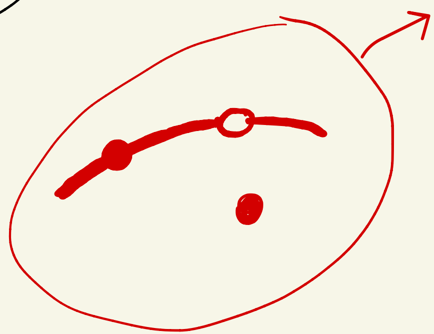
$c^{sm}(\Omega_{12} \subset \mathrm{Gr}_2\mathbb{C}^2) = 1$ $\quad\in H^*_T(\mathrm{Gr}_2\mathbb{C}^2)$

$$\boxed{\begin{aligned} c^{sm}(\Omega_{\emptyset} \subset Gr_0\,\mathbb{C}^2) &= 1 \\ c^{sm}(\Omega_{1} \subset Gr_1\,\mathbb{C}^2) &= z_2 - t \\ c^{sm}(\Omega_{2} \subset Gr_1\,\mathbb{C}^2) &= z_1 - t + \hbar \\ c^{sm}(\Omega_{12} \subset Gr_2\,\mathbb{C}^2) &= 1 \end{aligned}}$$

csm classes with "opposite" reference flag



vs



$$\boxed{\begin{aligned} c^{sm}(\Omega_{\emptyset} \subset Gr_0\,\mathbb{C}^2) &= 1 \\ c^{sm}(\Omega_{1} \subset Gr_1\,\mathbb{C}^2) &= z_2 - t + \hbar \\ c^{sm}(\Omega_{2} \subset Gr_1\,\mathbb{C}^2) &= z_1 - t \\ c^{sm}(\Omega_{12} \subset Gr_2\,\mathbb{C}^2) &= 1 \end{aligned}}$$

Calculation: change of basis matrix:

$$R(z_1 - z_2) := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{z_1 - z_2}{z_1 - z_2 + \hbar} & \frac{\hbar}{z_1 - z_2 + \hbar} & 0 \\ 0 & \frac{\hbar}{z_1 - z_2 + \hbar} & \frac{z_1 - z_2}{z_1 - z_2 + \hbar} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \frac{z_1 - z_2}{z_1 - z_2 + \hbar} \, \mathrm{Id} + \frac{\hbar}{z_1 - z_2 + \hbar} \, P$$

Fact: this matrix satisfies the

parameter-dependent Yang-Baxter equation:

$$R_{12}(z_1-z_2)\,R_{13}(z_1-z_3)\,R_{23}(z_2-z_3)=R_{23}(z_2-z_3)\,R_{13}(z_1-z_3)\,R_{12}(z_1-z_2)$$

(verify!!!)

meaning:

$\mathbb{C}^2 = \operatorname{span}(v_1, v_2)$ $\otimes\mathbb{C}(z_1,z_2)$

$$\begin{pmatrix} v_1\otimes v_1 \leftrightarrow c^{sm}(\Omega_\emptyset) \\ v_1\otimes v_2 \leftrightarrow c^{sm}(\Omega_1) \\ v_2\otimes v_1 \leftrightarrow c^{sm}(\Omega_2) \\ v_2\otimes v_2 \leftrightarrow c^{sm}(\Omega_{12}) \end{pmatrix}$$

$$\boxed{\mathbb{C}^2\otimes\mathbb{C}^2\otimes\mathbb{C}^2 \longrightarrow \mathbb{C}^2\otimes\mathbb{C}^2\otimes\mathbb{C}^2}$$

$R_{ij}(z)$ acts as $R(z)$ in i'th & j'th factor

$$
R_{12}(z_1 - z_2) = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{z_1 - z_2}{z_1 - z_2 + h} & 0 & \frac{h}{z_1 - z_2 + h} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{z_1 - z_2}{z_1 - z_2 + h} & 0 & \frac{h}{z_1 - z_2 + h} & 0 & 0 \\
0 & 0 & \frac{h}{z_1 - z_2 + h} & 0 & \frac{z_1 - z_2}{z_1 - z_2 + h} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{h}{z_1 - z_2 + h} & 0 & \frac{z_1 - z_2}{z_1 - z_2 + h} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{z_2 - z_3}{z_2 - z_3 + h} & \frac{h}{z_2 - z_3 + h} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{h}{z_2 - z_3 + h} & \frac{z_2 - z_3}{z_2 - z_3 + h} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{z_2 - z_3}{z_2 - z_3 + h} & \frac{h}{z_2 - z_3 + h} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{h}{z_2 - z_3 + h} & \frac{z_2 - z_3}{z_2 - z_3 + h} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} = R_{23}(z_2 - z_3)
$$

$$
R_{13}(z_1 - z_3) = \begin{array}{cl}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-z_3 + z_1}{z_1 - z_3 + h} & 0 & 0 & \frac{h}{z_1 - z_3 + h} & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-z_3 + z_1}{z_1 - z_3 + h} & 0 & 0 & \frac{h}{z_1 - z_3 + h} & 0 \\
0 & \frac{h}{z_1 - z_3 + h} & 0 & 0 & \frac{-z_3 + z_1}{z_1 - z_3 + h} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & \frac{h}{z_1 - z_3 + h} & 0 & 0 & \frac{-z_3 + z_1}{z_1 - z_3 + h} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
&
\begin{array}{l}
v_1 \otimes v_1 \otimes v_1 \\
v_1 \otimes v_1 \otimes v_2 \\
v_1 \otimes v_2 \otimes v_1 \\
v_1 \otimes v_2 \otimes v_2 \\
2 \quad 1 \quad 1 \\
2 \quad 1 \quad 2 \\
2 \quad 2 \quad 1 \\
2 \quad 2 \quad 2
\end{array}
\end{array}
$$

Further along this direction . . .

this particular solution of the YB equation is the R-matrix of $Y(gl_2)$

Yangian

$\longrightarrow$ $H^*_T\left(\bigsqcup_k Gr_k\mathbb{C}^n\right)$ is a $Y(gl_2)$-module

(with c^{sm}-classes playing the role of standard basis vectors)

"spin basis"