CSM classes

"Chern-Schwartz-MacPherson"

($\hbar$-deformed cohomological Schubert classes)

	12	13	14	23	24	34
$c^{sm}(\Omega_{12})$	$(z_3-z_1)(z_4-z_1)(z_3-z_2)(z_4-z_2)$	0	0	0	0	0
$c^{sm}(\Omega_{13})$	$\hbar \cdot \ldots$	$(z_2-z_1)(z_4-z_1)(z_4-z_3)(z_2-z_3+\hbar)$	0	0	0	0
$c^{sm}(\Omega_{14})$	$\hbar \cdot \ldots$	$\hbar \cdot (z_2-z_3+\hbar) \cdot \ldots$	$(z_2-z_1)(z_3-z_1)(z_2-z_4+\hbar)(z_3-z_4+\hbar)$	0	0	0
$c^{sm}(\Omega_{23})$	$\hbar \cdot \ldots$	$\hbar \cdot (z_2-z_3+\hbar) \cdot \ldots$	0	$(z_4-z_2)(z_4-z_3)(z_1-z_2+\hbar)(z_1-z_3+\hbar)$	0	0
$c^{sm}(\Omega_{24})$	$\hbar \cdot \ldots$	$\hbar(z_2-z_3+\hbar) \cdot \ldots$	$\hbar(z_2-z_4+\hbar)(z_3-z_4+\hbar) \cdot \ldots$	$\hbar(\ldots)(\ldots) \ldots$	$(z_3-z_2)(z_1-z_2+\hbar)(z_1-z_4+\hbar)(z_3-z_4+\hbar)$	0
$c^{sm}(\Omega_{34})$	$\hbar \cdot \ldots$	$\hbar(z_2-z_3+\hbar) \cdot \ldots$	$\hbar(z_2-z_4+\hbar)(z_3-z_4+\hbar) \cdot \ldots$	$\hbar(\ldots)(\ldots) \ldots$	$\hbar(\ldots)(\ldots)(\ldots) \ldots$	$(z_1-z_3+\hbar)(z_1-z_4+\hbar)(z_2-z_3+\hbar)(z_2-z_4+\hbar)$

$c^{sm}(\Omega_I)$ is the unique class in $H_T^*(Gr_k \mathbb{C}^n)[\hbar]$

- degree = dim (Gr) (deg $\hbar = 1$)
- $$c^{sm}(\Omega_I)\big|_I = \prod_{\substack{i\in I\\ j\in \bar{I}\\ i<j}} (z_j - z_i) \cdot \underbrace{\prod_{\substack{i\in I\\ j\in \bar{I}\\ i>j}} (z_j - z_i + \hbar)}_{c_I}$$
- $c^{sm}(\Omega_I)\big|_J$ divisible by $\hbar$ for $I \neq J$
- $c^{sm}(\Omega_I)\big|_J$ divisible by c_J

(• $c^{sm}(\Omega_I)\big|_J = 0$ if $J \nleq I$)

	12	13	14	23	24	34
$c^{sm}(\Omega_{12})$	$(z_3-z_1)(z_4-z_1)(z_3-z_2)(z_4-z_2)$	0	0	0	0	0
$c^{sm}(\Omega_{13})$	$\hbar(z_3-z_1)(z_4-z_1)(z_4-z_2)$	$(z_2-z_1)(z_4-z_1)(z_4-z_3)(z_2-z_3+\hbar)$	0	0	0	0
$c^{sm}(\Omega_{14})$	$\hbar(z_3-z_1)(z_4-z_1)(z_3-z_2+\hbar)$	$\hbar\cdot(z_2-z_3+\hbar)(z_2-z_1)(z_4-z_1)$	$(z_2-z_1)(z_3-z_1)(z_2-z_4+\hbar)(z_3-z_4+\hbar)$	0	0	0
$c^{sm}(\Omega_{23})$	$\hbar\cdot(\ )(\ )(\ )$	$\hbar\cdot(z_2-z_3+\hbar)\cdot(\text{--})(\ )$	0	$(z_4-z_2)(z_4-z_3)(z_1-z_2+\hbar)(z_1-z_3+\hbar)$	0	0
$c^{sm}(\Omega_{24})$	$\hbar\cdot\boxed{\text{MESS}}$	$\hbar(z_2-z_3+\hbar)(\ )(\ )$	$\hbar(z_2-z_4+\hbar)(z_3-z_4+\hbar)(\cdot)$	$\hbar(\text{--})(\text{--})(\ )$	$(z_3-z_2)(z_1-z_2+\hbar)(z_1-z_4+\hbar)(z_3-z_4+\hbar)$	0
$c^{sm}(\Omega_{34})$	$\hbar\cdot\boxed{\text{MESS}}$	$\hbar(z_2-z_3+\hbar)(\ )(\ )$	$\hbar(z_2-z_4+\hbar)(z_3-z_4+\hbar)(\cdot)$	$\hbar(\text{--})(\text{--})(\ )$	$\hbar(\cdot)(\cdot)(\text{--})$	$(z_1-z_3+\hbar)(z_1-z_4+\hbar)(z_2-z_3+\hbar)(z_2-z_4+\hbar)$

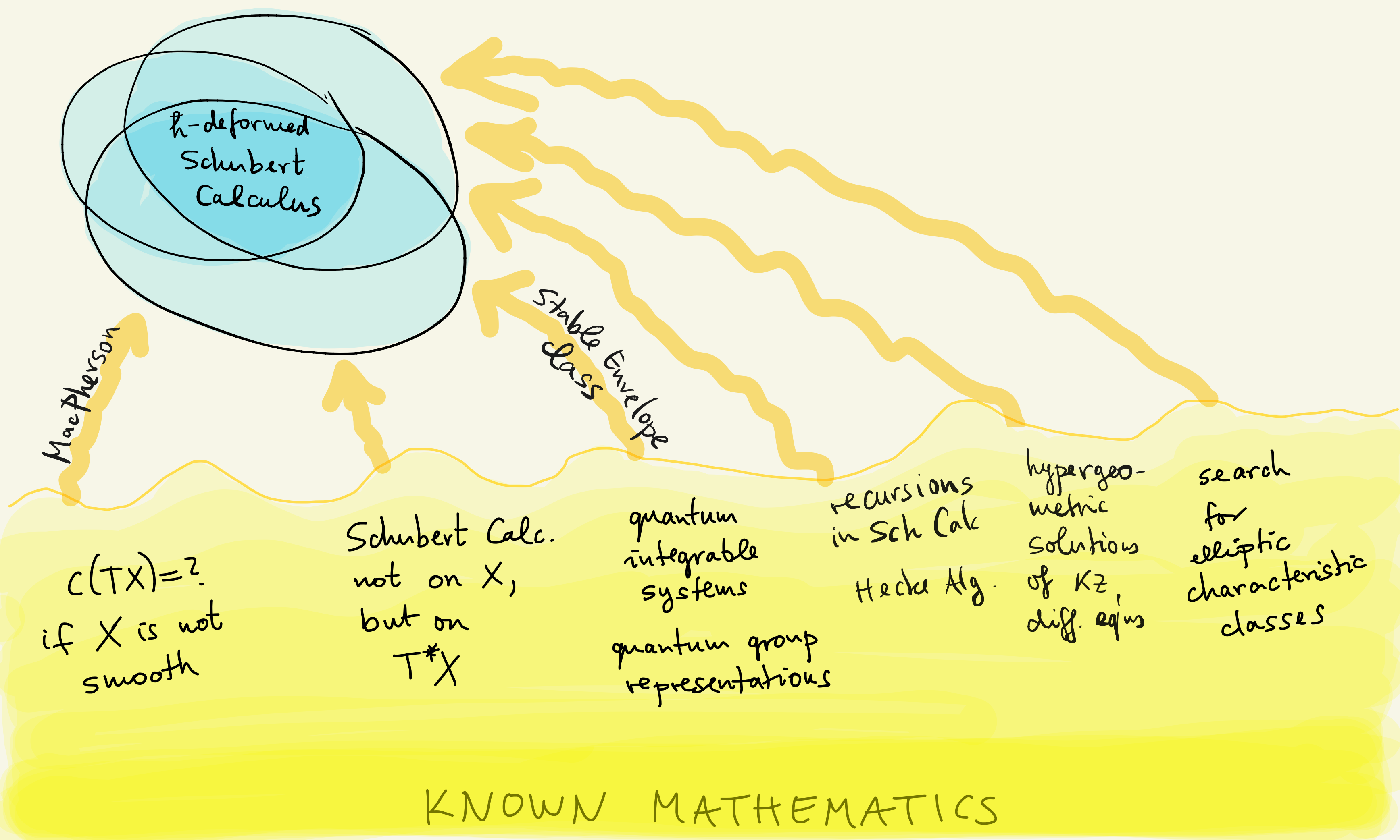
$\mathbb{P}^1$

	1	2	
$c^{sm}(\Omega_1)$	$(z_2 - z_1)$	0	$= z_2 - t$
$c^{sm}(\Omega_2)$	$?$	$(z_1 - z_2 + \hbar)$	$= z_1 - t + \hbar$

$\hbar \cdot (\deg\text{-}0) = \hbar \cdot A$

$\hbar A \;=\!=\; z_1 - z_2 + \hbar \implies A = 1$

when $z_1 = z_2$


ħ-deformed Schubert Calculus
MacPherson
Stable Envelope class
c(TX)=? if X is not smooth
Schubert Calc. not on X, but on T*X
quantum integrable systems
quantum group representations
recursions in Sch Calc
Hecke Alg.
hypergeo-metric solutions of KZ, diff. eq'ns
search for elliptic characteristic classes
KNOWN MATHEMATICS

Corrected vocab:

- what we called $c^{sm}(\Omega_I)$ is in fact $\mathrm{Stab}(\Omega_I)$

- $c^{sm}(\Omega_I) = \mathrm{Stab}(\Omega_I)\Big|_{\hbar=1}$

(same information, one determines the other)

Where are we?

$\mathrm{Gr}_k\mathbb{C}^n$

torus action

bundle

$\rightarrow H^*_T(\mathrm{Gr}_k\mathbb{C}^n)$

Schubert cells

Schubert class $[\bar{\Omega}_I] \in H^*_T(\mathrm{Gr}_k\mathbb{C}^n)$

$\hbar$-deformed Schubert class

CSM class $(\hbar = 1)$

stable envelope class

$c^{sm}(\Omega_I) \in H^*_T(\mathrm{Gr}_k\mathbb{C}^n)$

Rest of the talk :

illustrations of some properties of c^{sm} classes

- R – matrix property ... Yang - Baxter equation
- MacPherson point-of-view ... $c^{sm}(A \cup B)$
  c^{sm}(smooth compact)
- formulas ... weight functions ... Schur expansion
- why "cotangent" Schubert Calculus
- recursions
- how is c^{sm} "enumerative geometry"
- K theory version