Schubert classes

	12	13	14	23	24	34
$[\bar{\Omega}_{12}]$	$(z_3-z_1)(z_4-z_1)(z_3-z_2)(z_4-z_2)$	0	0	0	0	0
$[\bar{\Omega}_{13}]$	---	$(z_2-z_1)(z_4-z_1)(z_4-z_3)$	0	0	0	0
$[\bar{\Omega}_{14}]$	---	---	$(z_2-z_1)(z_3-z_1)$	0	0	0
$[\bar{\Omega}_{23}]$	---	---	0	$(z_4-z_2)(z_4-z_3)$	0	0
$[\bar{\Omega}_{24}]$	---	---	---	---	(z_3-z_2)	0
$[\bar{\Omega}_{34}]$	---	---	---	---	---	1

Schubert classes

Thm-Def

$[\bar{\Omega}_I]$ is the unique class in $H_T^*(Gr_k \mathbb{C}^n)$

- degree $= \#\{(i,j) : i \in I, j \in \bar{I}, j > i\}$

- $[\bar{\Omega}_I]\big|_I = \prod_{\substack{i \in I \\ j \in \bar{I} \\ j > i}} (z_j - z_i)$

$J \leq I$

$J = \{j_1 < j_2 < \dots < j_k\}$

$\wedge\!\!| \quad \wedge\!\!| \quad \cdots \quad \wedge\!\!|$

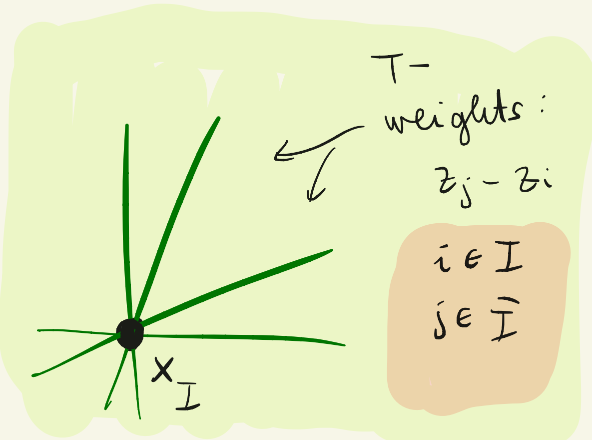
$I = \{i_1 < i_2 < \dots < i_k\}$

- $[\bar{\Omega}_I]\big|_J = 0$ if $J \not\leq I$

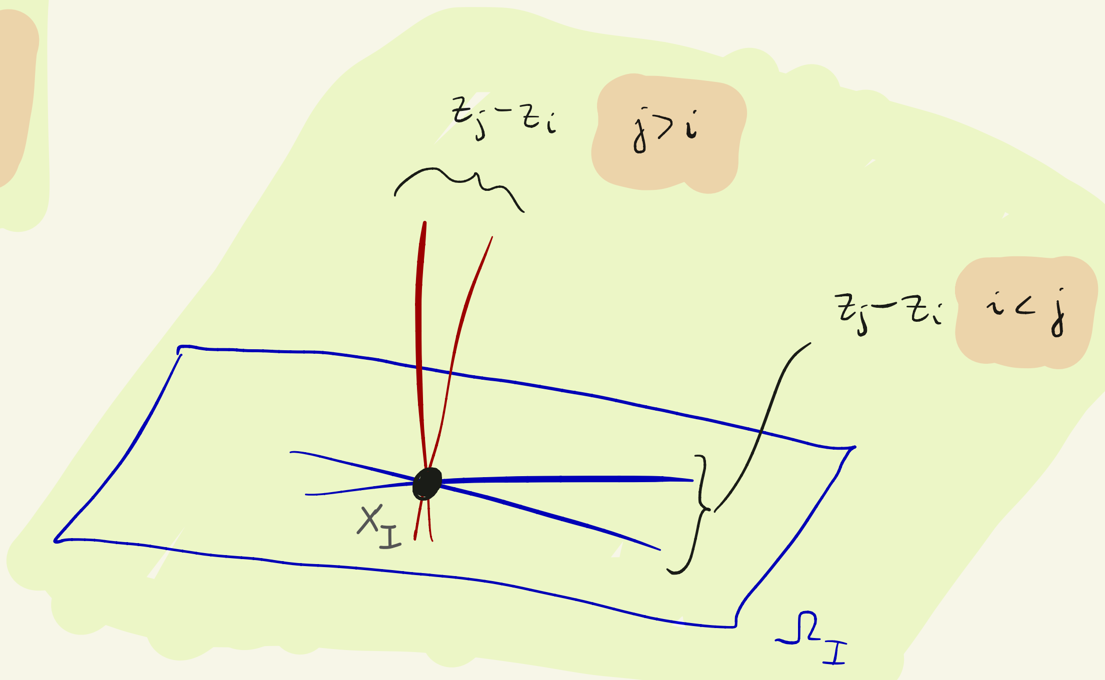
"axiomatic characterization of Schubert classes"

	12	13	14	23	24	34
$[\bar{\Omega}_{12}]$	$(z_3-z_1)(z_4-z_1)(z_3-z_2)(z_4-z_2)$	0	0	0	0	0
$[\bar{\Omega}_{13}]$	$(z_3-z_1)(z_4-z_1)(z_4-z_2)$	$(z_2-z_1)(z_4-z_1)(z_4-z_3)$	0	0	0	0
$[\bar{\Omega}_{14}]$	$(z_3-z_1)(z_4-z_1)$	$(z_2-z_1)(z_4-z_1)$	$(z_2-z_1)(z_3-z_1)$	0	0	0
$[\bar{\Omega}_{23}]$	$(z_4-z_1)(z_4-z_2)$	$(z_4-z_1)(z_4-z_3)$	0	$(z_4-z_2)(z_4-z_3)$	0	0
$[\bar{\Omega}_{24}]$	$z_4+z_3-z_1-z_2$	z_4-z_1	z_3-z_1	z_4-z_2	(z_3-z_2)	0
$[\bar{\Omega}_{34}]$	1	1	1	1	1	1

Geometric explanation of diagonal restrictions



$Gr_k \mathbb{C}^n$ around fixpoint x_I



recall

$$H_T^*(Gr_2\mathbb{C}^4) = \begin{cases} \{ (f_{12}, \dots, f_{34}) \ : \ \dots \} \\ \{ (f_{12}, \dots, f_{34}) \ : \ \exists f(t_1, t_2, \dots) \ \dots \} \end{cases}$$

	12	13	14	23	24	34
$[\Omega_{12}]$	$(z_3-z_1)(z_4-z_1)(z_3-z_2)(z_4-z_2)$	0	0	0	0	0

$t_1 = z_1, \ t_2 = z_2$

$t_1 = z_1, \ t_2 = z_3$

$t_1 = z_3 \quad t_2 = z_4$

$$(z_3 - t_1)(z_4 - t_1)(z_3 - t_2)(z_4 - t_2)$$

	12	13	14	23	24	34
$[\bar{\Omega}_{13}]$	$(z_3-z_1)(z_4-z_1)(z_4-z_2)$	$(z_2-z_1)(z_4-z_1)(z_4-z_3)$	0	0	0	0
	$t_1=z_1$, $t_2=z_2$	$t_1=z_1$, $t_2=z_3$	$t_1=z_1$, $t_2=z_4$			$t_1=z_3$, $t_2=z_4$

$$\frac{(z_2-t_1)(z_3-t_1)(z_4-t_1)(z_4-t_2)}{(t_2-t_1)} + \frac{(z_2-t_2)(z_3-t_2)(z_4-t_2)(z_4-t_1)}{(t_1-t_2)}$$

$t_1 \leftrightarrow t_2$

- polynomial (easy algebra)(concrete simplified form is not important)

In general, $Gr_k \mathbb{C}^n$, $I = \{ i_1 < i_2 < \dots < i_k \}$

$$[\bar\Omega_I] = \operatorname*{Sym}_{t_1 \dots t_k} \left(\prod_{a=1}^{k} \prod_{b=i_a+1}^{n} (z_b - t_a) \cdot \prod_{1 \le a < b \le k} \frac{1}{t_b - t_a} \right)$$

- polynomial
- satisfies the axioms

$$[\bar\Omega_I]\big|_I = \prod_{\substack{i \in I \\ j \in \bar I \\ j > i}} (z_j - z_i)$$

$$[\bar\Omega_I]\big|_J = 0 \quad \text{if} \quad J \not\leq I$$

Rem substituting $z_i = 0$ the formula simplifies to

$$\det \begin{pmatrix} t_1^{n-i_1} & t_2^{n-i_1} & t_3^{n-i_1} & \dots \\ t_1^{n-i_2} & t_2^{n-i_2} & t_3^{n-i_2} & \dots \\ \vdots & & & \end{pmatrix} \Big/ \det \begin{pmatrix} 1 & t_1 & t_1^2 & \\ 1 & t_2 & t_2^2 & \\ 1 & t_3 & t_3^2 & \dots \\ \vdots & \vdots & \vdots & \end{pmatrix}$$

Schur polynomial

Where are we so far

- $H^*_T(Gr_k\mathbb{C}^n)$ description $= \{ (f_I) : \ldots \}$
- $[\bar{\Omega}_I] \in$

defined by interpolation axioms

by $f(t_1 \ldots t_k, z_1 \ldots z_n)$ formula

Coming up

$\hbar$-deformation of $[\bar{\Omega}_I]$