Schubert cells

$Gr_2\mathbb{C}^4$

fix $\mathbb{C}^1 \subset \mathbb{C}^2 \subset \mathbb{C}^3 \subset \mathbb{C}^4$ "reference flag"

$\Omega_{12} = \{V :$	$\dim(V\cap\mathbb{C}^1) = 1,$	$\dim(V\cap\mathbb{C}^2) = 2,$	$\dim(V\cap\mathbb{C}^3) = 2,$	$\dim(V\cap\mathbb{C}^4) = 2\}$
$\Omega_{13} = \{V :$	1	1	2	$2\}$
$\Omega_{14} = \{V :$	1	1	1	$2\}$
$\Omega_{23} = \{V :$	0	1	2	$2\}$
$\Omega_{24} = \{V :$	0	1	1	$2\}$
$\Omega_{34} = \{V :$	0	0	1	$2\}$

$$Gr_2\mathbb{C}^4 = \Omega_{12} \cup \Omega_{13} \cup \Omega_{14} \cup \Omega_{23} \cup \Omega_{24} \cup \Omega_{34}$$

$$\mathrm{Gr}_2\mathbb{C}^4 = \Omega_{12} \cup \Omega_{13} \cup \Omega_{14} \cup \Omega_{23} \cup \Omega_{24} \cup \Omega_{34}$$

Ω_{12}: pt

Ω_{13}: dim = 1

Ω_{14}: dim = 2

Ω_{23}: dim = 2

Ω_{24}: dim = 3

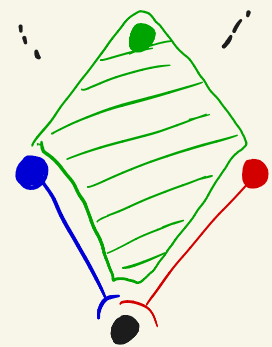
Ω_{34}: dim = 4

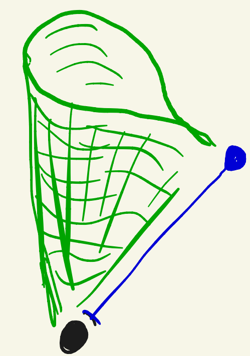
- Ω_{ij} cell $\cong \mathbb{C}$[redacted]

  Schubert cell

- $\overline{\Omega}_{ij}$ Schubert variety

  in general not smooth





Schubert decomposition

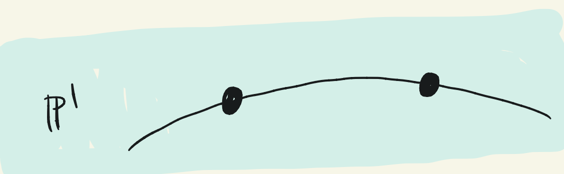
fix $\mathbb{C}^1 \subset \mathbb{C}^2 \subset \dots \subset \mathbb{C}^n$

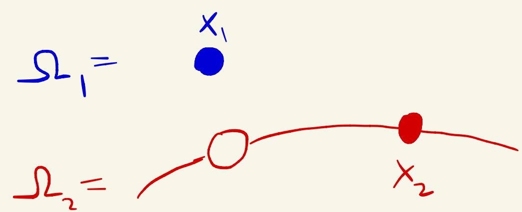
"reference full flag"

$I \subset \{1 \dots n\}$

$|I| = k$

$$x_I \in \quad \Omega_I := \left\{ V \in Gr_k \mathbb{C}^n : \dim(V \cap \mathbb{C}^q) = |\{ i \in I : i \leq q \}| \right\}$$





Schubert decomposition induces a **partial order** on

Schubert-cells $\longleftrightarrow$ fixpts $\longleftrightarrow$ k-element subsets of $\{1 \dots n\}$

$\Omega_I \geq \Omega_J$

if

$\overline{\Omega_I} \supseteq \Omega_J$

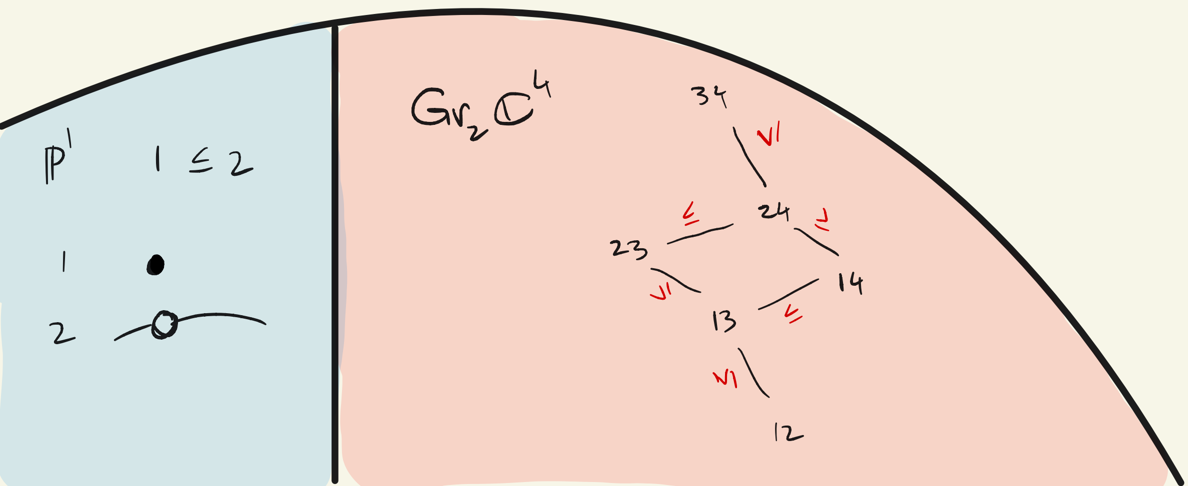
$I = \{ i_1 \leq \dots \leq i_k \}$

$J = \{ j_1 \leq \dots \leq j_k \}$

$I \geq J$

if

$i_r \geq j_r \quad \forall r$



Where are we so far

$Gr_k \mathbb{C}^n$

- torus action
- bundle
- Schubert cells

$\rightarrow H^*_T(Gr_k \mathbb{C}^n)$

explicit description

Coming up

Schubert cell $\longrightarrow$ $\boxed{\text{“its class”}}$ $\in H^*_T(Gr_k \mathbb{C}^n)$

two versions:

- $[\overline{\Omega}_I]$ Schubert class
- $c^{sm}(\Omega_I)$ CSM class